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A study of changes in students' understanding of three algebraic concepts: variables, expressions, and equality

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A study of changes in students' understanding of three algebraic concepts: variables, expressions, and equality

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Dedication

I dedicate this work to my mother, Clare Corrington Brice, who led by example. A driving force in my life is my husband, Randy Cushman. Cheri McCullough, Debbie Junk, Sarah Davis, Tony Petrosino, and Luptia Carmona: thank you for your invaluable support and constructive criticism. The biggest "Thank you" goes to my friend, Jenn C. Smith. She gave me her guidance and patience without which I would not have succeeded.

A study of changes in students' understanding of three algebraic concepts: variables, expressions, and equality

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This study of ASK ME-Algebra, an online high school algebra curriculum, focuses on students acquiring knowledge about the concepts of variables, expressions, and equality. The Online Mathematics Initiative (OMI) of the Distance Education Center (DEC) of the University of Texas researched and developed the algebra curriculum to address and encompass both Texas's and national standards for high school algebra. Twelve school districts in Texas used ASK ME-Algebra in classrooms and with independent learners for credit recovery[1]. Designed by researchers as a functions-based and problem-based instructional approach to algebra, ASK ME-Algebra assists students

[1] Credit recovery is for students who have failed a course. The students take the course again but in a shorter time-period (i.e. one semester instead of a whole school year or two semesters).

to gain an understanding of algebra concepts (i.e. functions, linear functions, linear equations and systems of equations). The methodology chosen for this study was a design experiment since it aligns with the constructivist viewpoint of the researcher. This design experiment will show that ASK ME-Algebra increases students' understanding of the concepts of variables, expressions, and equality without structured instruction. The teacher of the classroom negatively affected the implementation of the curriculum by using structured instruction during the last two-thirds of the course. The data for this study includes responses to pre- and post-algebra tests, comparative data from ninth grade Algebra I students, observations, all of the treatment groups' saved online work, and periodic interviews of the treatment group from a central Texas alternative high school using the online curriculum as independent learners.

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Chapter 1 Introduction

"We know that students leave school for many reasons other than just moving away: Some don't accumulate enough credits to pass to the next grade with their peers, and become frustrated; *algebra is too often a stumbling block*; sometimes personal or family issues overwhelm academic goals; and students who leave say they'll just get a General Equivalency Diploma (GED) instead." [emphasis added] (Tom Craddick, 9/15/2004)

Through experiences as a graduate student and an instructor, I developed two research questions. The first question is: In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' understanding of concepts, such as variables, equality, and equivalent expressions? The second question is: In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics? A discussion of the background information about the experiences that lead to the questions occurs in this chapter.

Problem Addressed by This Study

All ninth grade students in the state of Texas are required to take Algebra I[2]. Unfortunately, a number of students (many Latino/a and low SES) across the state are not successful in receiving credit for the course and cannot graduate from high school

[2] This requirement was implemented in the fall of 1994 by the State of Texas legislature.

without passing algebra. Finding a successful method of teaching algebra to all students was the impetus for my work in this study. If students understand the concepts of variables, expressions, and equality through any curriculum then perhaps the instructional method described in this study was successful.

Project-Based Instruction

After teaching high school algebra for 15 years and using a state-adopted textbook with an algorithmic approach, many of my students did not understand algebraic concepts, based on the students' scores on the relevant performance measures[3]. The impetus for beginning my pursuit of a doctorate was to find a "new and improved" method of instruction in algebra for *all* students.

During my first year at The University of Texas at Austin (UT) and working with the UTeach program[4], I learned of a new instructional method that develops understanding: Project-Based Instruction (PBI) (Blumenfeld, Soloway, Marx, Krajcik, Guzdial and Palincsar, 1991; Boaler, 1997; Wiggins & McTighe, 1998). The PBI class had three concentrations: students observed a PBI high school class, taught a PBI lesson to high school students at the UT Marine Science Institute, and developed a PBI six-week-long project for implementation in their future classrooms. The PBI project included a guiding question, an anchor video, a project syllabus, lesson plans, and assessment tools. My observation of and participation in the PBI classes, lead to one of two research questions for this dissertation study: In what ways could ASK ME-Algebra

[3] End Of Course (EOC) Algebra test; their final grades for the course

(or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics?

A traditional approach in high school mathematics includes simple skills taught before students apply their knowledge to a complex situation (Boaler, 1997; Wiggins & McTighe, 1998; Blumenfeld, Krajcik, Marx, & Soloway, 1994). In contrast, in a PBI environment, students are given realistic problems, gather additional information, create solutions, and present their solutions. Research indicates that a PBI approach to learning benefits students in several ways (Blumenfeld et al, 1991; Delisle, 1997; Heid & Zbiek, 1995). Since multipart problems serve as the framework for students to integrate different kinds of knowledge and skills, students learn algebraic content, mathematical reasoning, and a means of managing their own learning (Kilpatrick, 1987; Cobb, Wood, & Yackel, 1992). Realistic problem solving leads to enduring understanding (Wiggins and McTighe, 1998). According to Boaler (1997), real-world situated open-ended problems can grasp the attention of students and keep them motivated until they find an answer. Realistic problems are open-ended and there are multiple final solutions; a student's answer can be in a Word document, in a PowerPoint presentation, and even on a poster. Here is an example: "Write a new speeding fine structure for Cartwaller

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[4] UTeach is a secondary certification opportunity for undergraduates and post-baccalaureates who major in mathematics, science, or computer science.

Avenue. Remember that the total fine for going 10 miles per hour over the speed limit should be between $130 and $140. And, according to the city council, going more than 25 miles per hour over the speed limit would mean at least a $200 fine."

A teacher could read a letter from the city council asking for a tougher speeding fine structure with the above restrictions, which will help scaffold student's first attempt at an open-ended problem. After exploring other fine structures (linear, piece-wise, and a combination of the two), the students can develop their speeding fine structure and a presentation to the city council explaining why their speeding fine structure meets the restrictions. One assessment possibility is to have the students present their speeding fine to a city council member while the teacher and/or council member uses a rubric to assign a grade for the presentation and the mathematics involved.

Online Mathematics Initiative

The Distance Education Center (DEC) of UT wanted to change its correspondence course to an online format. After introducing the idea to the Texas Governor's office, the Online Mathematics Initiative (OMI) started in the fall of 2001 following the approval of the biennial budget by Texas legislature. The OMI developed a curriculum for teaching and communicating about mathematics online; high school algebra was the first course. While working with the OMI, I learned about another approach to teaching algebra, known as a *functions-based* approach to algebra. Most students increase their understanding of algebraic concepts effectively with a functions-based algebra approach (Heid et al, 1995; Malloy & Malloy, 1998; NCTM, 1999). Learning about this new approach to teaching and organizing algebra lead to the second

research question for this dissertation study: In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' understanding of concepts, such as variables, equality, and equivalent expressions?

Functions-Based Algebra

The state-adopted textbooks used in most Texas school districts, focus on *an algorithmic* algebra (Li, 2005), and do not lead many students to understand algebraic concepts[5]. The students became proficient with skills: using order of operations, simplifying expressions, and solving basic equations. Many high school teachers in Texas use some form of technology (usually graphing calculators) in mathematics class since each school district must provide a graphing calculator to each student during the mathematics and science portions of TAKS tests (Texas Education Agency, 2006).

A functions-based algebra curriculum uses letters primarily to denote variables that represent dynamic changes in functional relationships. The emphasis of the curriculum is with modeling and realistic problems. Students determine what are the variables in a problem situation, their interdependence, formulate function rules, and make predictions using multiple representations (verbal, symbolic, tabular and graphic) (Li, 2005).

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[5] This conclusion is based on Texas students' scores on the End-of-Course (EOC) algebra test. Many students did not receive passing grades in algebra classes or on the EOC algebra test. The high failure rate of the EOC algebra test can be attributed to many things, not the least of which was that the test did not have any consequences for students. The state of Texas retained the data from the results of the EOC algebra test; the test did not prevent students from graduating from high school or from receiving credit in algebra. Many students in the state of Texas did not seem to understand the algebraic concepts tested in

The algebra curriculum developed by the Online Mathematics Initiative depends on various uses of technology; therefore, this study considers the role technology plays in learning. For instance, Shockwave video and Flash animation applications provide opportunities for students to interact with the mathematical content. For example, these technologies allow students to generate an unlimited number of practice problems. Students can create graphics and submit them to their instructors electronically and proper mathematical notion is easily used. Technology (e.g. computers with graphing software and a spreadsheet tool) has been shown to help to improve students' understanding of algebraic concepts, especially linear equations (Pugalee, 2001; Yerushalmy, 2004). In Pugalee's study, students would generate conjectures about sets of linear equations and then verify their assumptions using their graphing calculators. Yerushalmy describes how students learned to solve systems of equations graphically before learning any symbolic methods because the students were using graphing calculators.

ASK ME-Algebra

The DEC started the Online Mathematics Initiative (OMI) in the fall of 2001. OMI initially focused on supporting the development of functions-based algebra curricula. Researchers Cathy Seeley (2004 - 2006 NCTM president) and Diane McGowan (Charles A. Dana Center Mathematics specialist) were part of the initial team.

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algebra. Only 45% of all algebra students in Texas passed the EOC algebra test in 1999 (Texas Education Agency, 1999).

Joey Offer, an instructional designer and the director of the project, invited me to become involved with OMI in the spring of 2002 as a graduate research assistant[6].

The team built upon the work of Bransford, Brown, and Cocking (2000) and Wiggins and McTighe (1998) to develop the curriculum[7] by using student-centered, community-centered and assessment-centered problems. The first unit, "An Introduction to Functions", was written around the theme of a community trying to curb speeding through its neighborhoods and was written to emphasize the central concept of the curriculum: the function. The realistic problem requires students to determine the independent variable, the dependent variable, a function rule, and to represent the outcome with multiple representations.

At the same time, the DEC's staff of technology specialists was developing a web-based program to allow distance-learning courses to be completed online. Until recently, correspondence courses were still textbook-based (i.e. distance education students would do assigned work out of the textbook and mail in their correspondence assignments then wait two weeks or so for the graded assignment and feedback from the instructor). Currently, distance education students are able to submit their assignments electronically and receive feedback and grades within 48 hours. The DEC staff also developed application tools that would allow the following:

1. Students and teachers communicate mathematically with symbols, tables, equations, and graphs.

[6] I am also a certified Texas secondary mathematics teacher. This was advantageous later when we tested the material of the first unit in a face-to-face setting with students.

2. Simulated manipulatives help students understand abstract concepts (i.e. algebra tiles).
3. An online graphing calculator allows students to further explore concrete representations of abstract concepts (Offer, Seeley, Williams, Lee, Ries and Nankervis, unpublished)

The course was designed to actively engage students in their own learning even while they were working by themselves and at their own pace using a computer on the Internet. Students interact with a certified teacher when needed by intranet email. The ASK ME—Algebra course is focused around four important goals, each of which was supported through the use of technology:

- Focusing on a functions-based approach to algebra
- Connecting algebra to realistic problems
- Addressing the acquisition of skills
- Supporting transfer of learning to future situations

The multimedia environment also assists students in learning. Each of the three units has an anchor video with four animated characters that call themselves the "Cartwaller Algebra Society". The algebra society is a community of learners and the animated characters model the kind of mathematical thinking and communication that we expect from the students. The animated characters also form a community of learners with each student by asking the student questions and expecting responses, and the

[7] See Appendix F for the Scope and Sequence of ASK ME-Algebra.

characters help the student to learn important concepts by being the "knowledgeable others" the student can turn to repeatedly.

The tutorials provide more structure than other components of the course. The tutorials involve active learning and the modeled feedback students receive enables students to modify their own work. Many tutorials allow unlimited practice.

The design of each unit in the algebra course centers on a realistic community problem. For instance, to explore linear functions the students determine how much the community leaders need to charge citizens for collecting garbage and recycling in their community. The students develop an algebraic rule that fits the criteria from the community leaders. This design follows guidelines of Problem-Based Instruction (PBI): students use mathematical communication, i.e. students explain and justify, and have opportunities to make conjectures (Brenner et al. 1997). The structure of the community problems enables students to apply algebra in a realistic context. The most of the questions in the activities and the graded assignments are open-ended to afford students opportunities to experience algebra as a coherent body of knowledge and tools rather than a set of rules that produce one correct answer (Offer et al., unpublished). In the activities, the feedback given to each student models the kind of mathematical thinking and communication expected from the students. In order to elicit "enduring understanding" from the students, the questions, designed from the work summarized in Wiggins and McTighe (1998), ask for in-depth, revealing and skilled thinking.

> ASK ME—Algebra uses technology in service to the teaching, learning and assessing of mathematics. The many ways technology

> is used make the program interactive and unique; the mathematics makes the program strong and grounded. The combination of the two makes the course accessible, engaging, and academically useful (Offer et al., unpublished, p. 2).

The target audience for ASK ME-Algebra when it was in development was ninth grade Texas students in an independent environment. An ideal scenario follows.

A ninth grade student and his or her parents decide that the student could learn algebra at his or her own pace and enrolls in the first semester of the ASK ME-Algebra course. Since the family owns a computer and has an Internet connection, they feel the student is ready to try an online course. The student, intrigued by the instructional approach and the use of the medium, is motivated to work, watch, and read all of the content. The student starts by watching the anchor video for Unit 1. Next, the student interacts with the tutorial in the first Activity. The student attempts to answer each question in the Guided Practice and saves his or her work. Next, the student checks his or her work by reading each explanation. This process repeats for each activity. The student feels prepared for the first Graded Assignment. The student notices the multiple choice items in the Graded Assignment are similar to the questions on the TAKS test he or she has been taking since third grade. The student notices a link for a "rubric" on each question in the Graded Assignment. Just out of curiosity, the student views the rubric for the first problem. He or she discovers that the rubric is a guide to answering the problem.

As long as the student addresses each item in the rubric, his or her answer will be complete. After sending in his or her first graded assignment, the student learns that the instructor is a resource; someone he or she can ask when the student is not sure what a problem is asking. After redoing a question or two from his or her first graded assignment, the student begins to use all of the resources in the course: the glossary, the tutorial index, the instructor, and the explanations in the Guided Practice. In the second Graded Assignment, the student is asked to make a scatterplot of given data. The student is comfortable with using his or her graphing calculator to generate the scatterplot. The student notices that a Graded Assignment has a link to an application called MathPad. The student realizes the MathPad is similar to his or her graphing calculator and allows him or her to send scatterplots, functions, tables, and other items to the instructor to evaluate. In a timely manner, the student works through all of the curriculum and with the guidance of the instructor and is successful at understanding the concepts in the first semester. He or she takes the final exam and receives credit for the course.

In conclusion, after experiences as a teaching assistant with a PBI class, as a graduate research assistant with the OMI, and as an employee with the ASK ME-Algebra project, the following questions were developed for this dissertation. The research questions addressed in this study, in order of importance, are:

- In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' understanding of concepts, such as variables, equality, and equivalent expressions?

- In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics?

Addressed in the next chapter is the relevant research on variables, expressions, and equality along with students' misconceptions and what research shows can help students understand the concepts of variables, expressions, and equality. Also discussed is the view students' hold about mathematics. The main views are: students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics. The third chapter will discuss the methods, the participants, and the instruments utilized while conducting this study. The results and analysis per student are reported in the fourth chapter. The fifth chapter looks at the interpretations of the data as a whole. The final chapter discusses the limitations of this study and the future possibilities of research.

Chapter 2 Literature Review

This chapter contains an overview of the algebraic concepts that research has shown students struggle with the most when first learning algebra. Also included is a discussion of what researchers have shown are ways to overcome these "stumbling blocks" to students' understanding of algebra. A discussion of the beliefs many students hold about mathematics also occurs.

National Algebra Standards

There are no official national mathematical standards for the United States of America, but the National Council of Teachers of Mathematics (NCTM)'s standards for school mathematics is considered a model. NCTM developed the *Curriculum and Evaluation Standards for School Mathematics* in 1989. It revised the 1989 standards and developed the *Principles and Standards for School Mathematics* (*PSSM*) in 2000. The *PSSM* are divided into content standards (including algebra) and process standards (including communication). The algebra strand for Pre-Kindergarten to 12th grade from the *PSSM* has four main components:

- Understand patterns, relations, and functions;
- Represent and analyze mathematical situations and structures using algebraic symbols;
- Use mathematical models to represent and understand quantitative relationships; and
- Analyze change in various contexts (NCTM, 2000, p. 296)

These components are addressed at increasing levels of difficulty through all of the grade levels, Pre-K through 12th grade. Algebra is the strand that is to receive the most focus for 9th –12th grades.

The process standards for the content strand of algebra include: problem solving, reasoning and proof, communication, connections, and representations. These process standards expand on the content of each subcategory of algebra. For example, solving linear equations using multiple representations, including tables, graphs and symbols, can help students to understand the relationships between and among the different representations of a linear equation.

The *PSSM* help teachers decide what is taught in the classroom, but many students will struggle with basic concepts like variables, expressions, and equality.

Students' misconceptions of algebra

The concepts that students need to understand before working with linear equations are variables, algebraic expressions on each side of the equal sign, and relational equality. A complete understanding of each part of the linear equation will help students to determine which method (symbolic, graphic, or tabular) will allow them to solve any linear equation they are faced with throughout their lives (Kuchemann, 1978; Kieran, 1981; MacGregor & Stacy, 1997; Kieran & Sfard, 1999).

Variables

One of the misunderstandings students often bring to algebra class relates to the concept of variables[8] (Graham & Thomas, 2000; Kuchemann, 1978; Pugalee, 2001). For example, when x is multiplied by 7 and written as 7x, many students infer this to mean x is the unit digit of a two-digit number and even other students think that 7 and x are added together in a similar manner as integers and fractions are added together in a mixed number (MacGregor & Stacy, 1997). Another difficulty students have with variables occurs when students are told that variables represent numbers (known and unknown) (MacGregor & Stacy, 1997). Unfortunately, students see letters used to represent other things outside of algebra class (e.g., p. 6 for page 6, e which represents a particular number, and $\angle ABC$ where A, B, and C represent vertices). Even teachers will add to the confusion by making statements like, 'Let C denote the circumference,' so some students will infer the letter C means the word circumference (MacGregor & Stacy, 1997). A specific case reported by McGregor & Stacy (1997) showed that 15 year-old students believed that x was equal to 1. Some students even thought that any letter alone was equal to 1. This could come from teachers saying, 'x without a coefficient means 1x' or '$x^0 = 1$'. As Chazan & Yerushalmy found, "the meaning of variable is variable" (NCTM, 2000, p. 125). Not having a single meaning for *variable* can lead to confusion on the students' part.

[8] The definition of a variable from Merriam-Webster's online dictionary (www.m-w.com) states that a variable is "a quantity that may assume any one of a set of values" and "a symbol representing a variable".

Understanding Variables

The many ways in which variables are used can lead to confusion for students when working with another important concept in algebra: the algebraic expression (Kieran & Sfard, 1999). Students need a solid grasp of the idea of what a variable is in order to be successful with algebraic expressions. One way to ensure students grasp what a variable is, would be to introduce students explicitly to the many ways variables are used: "as names for objects, as discrete unknowns, as continuous unknowns in inequalities, as indeterminates in polynomials, as generalized numbers in identities, as independent and dependent variables in functions, and as parameters in formulas" (Wagner & Parker, 1993, pp. 129). According to Graham & Thomas (2000), "the idea of a variable is in fact a key concept in algebra – although many elementary texts do not explain or even mention it" (pp.265); students will make up their own understanding of the word *variable* when the meaning of *variable* is not clear to them. Variables are used in many ways, but students do not connect these ways together on their own.

> Usiskin (1989) offers the following:
>
> Consider these equations, all of which have the same form–the product of two numbers equals a third:
>
> 1. $A = LW$
> 2. $40 = 5x$
> 3. $\sin x = \cos x * \tan x$
> 4. $1 = n * \frac{1}{n}$
> 5. $y = kx$
>
> Each of these has a different feel. We usually call (1) a formula, (2) an equation (or open sentence) to solve, (3) an identity, (4) a

> property, and (5) an equation of a function of direct variation (not to be solved). These different names reflect the different uses to which the idea of variable is put. (p. 9)

Teachers should ensure that all school children understand the different uses of variables. This emphasis on the variety of ways that variables are used can deter students from developing their own, possibly incorrect, understandings (Usiskin, 1989).

Algebraic expressions

Simplifying algebraic expressions is a topic most students will encounter after trying to grapple with the concept of a variable. Kieran and Chalouh (1993) state that "the incongruencies between arithmetic and algebra and the consequent inability of novice algebra students to regard algebraic expressions as legitimate answers" are barriers to learning (p 43). In a study in which beginning algebra students completed tasks involving expressions, it was found that the students' methods were filled with unsystematic errors. The researchers concluded that the students had an absence of knowledge about the structure of algebra. Two other conclusions drawn from this study were that beginning algebra students are not consistent in their approach before performing an operation, and that the students are not consistent in their performance of the operations themselves. Kieran and Chalouh (1993) note that students just starting algebra have many misconceptions with expressions using variables, operations, and numbers; not understanding the structure of an algebraic expression can lead to many errors on the part of the students.

In the NCTM publication, *Algebra in a Technological World*, Heid, Choate, Sheets, & Zbiek (1995) comment:

> Students too often leave their algebra experience with a modicum of ability to produce equivalent forms but very little understanding of the meaning of that equivalence. In a technological world in which students have access to computer-algebra utilities [...] the importance of producing equivalent forms no longer overshadows the importance of understanding what the equivalent expressions mean. (p. 127)

NCTM advocates that students learn concepts "in depth," with the implication being that students need to fully understand the concepts. When students completely understand how to generate equivalent expressions, then students are far more likely to succeed with other algebraic concepts (NCTM, 1999).

Understanding Algebraic Expressions

The following lesson description from Heid & Edwards (2001) can help most students gain an understanding of expressions. Using Computer Algebra System (CAS)[9], students explored algebraic expressions. Expanding on the standard box problem, algebra students studied properties of a box they made by cutting square corners from rectangular pieces of cardboard that measured 14 x 8 square units. Students found the area of the remaining cardboard by adding together the different areas. They concluded

[9] Software embedded in some graphing calculators.

that the surface area (sa) of the box could be written in the following forms, which the students discovered to be equivalent:

1. sa = 2x * (8-2x)+2x * (14-2x)+(14-2x) * (8-2x)
2. sa = 8 * (14-2x)+2x * (8-2x)
3. sa = 14 * (8-2x)+2x(14-2x)

During several class periods, students discussed, generated and investigated the equivalence of the above expressions using a calculator that includes Computer Algebra System (CAS): Texas Instruments TI-92. For each expression entered, the calculator responded with $-4x^2 + 112$. The exercise encouraged a discussion about the distributive property.

Students who were generally unwilling to talk about algebraic processes were curious about the equivalence of the surface area expressions. Most of the students wanted to know why the expressions were equivalent. The activity promoted discussion of algebraic distribution, with students and instructors actively participating in the discourse.

An extension of the lesson used the multi-representational abilities of the TI-92; students used the graphing feature of the TI-92 to determine the maximum volume of the box, as well as the surface area of the box with maximum volume. The context of the problem made it worth working with abstract algebraic concepts, such as domain, range, and graphical representations (Heid & Edwards, 2001).

Without the technology of the CAS, this lesson would be long, dull and very tedious for most students. The technology of CAS helps keep students sufficiently

engaged in the lesson and to realize that the expressions are equivalent (Heid & Edwards, 2001; Kohol-Voljc, 1999; Graham & Thomas, 2000). With CAS, algebra can become multi-representational and less algorithmically focused. Students who have a conceptual understanding of equivalent expressions are ready to understand equations and how solve equations of many types with multiple methods.

With the use of CAS, the teaching and learning of secondary mathematics should change from memorizing and replicating algorithms to problem solving and exploring concepts. Teachers of mathematics can back away from having students "perform" mathematics and move towards students constructing their own understanding of the content and applying their new understanding. Students should not merely manipulate algebraic expressions, but they should be given tasks that require an understanding of why the mathematical processes they have learned work and when to apply them. For example, Kokol-Voljc (1999) describes an activity involving the prime factorization of 10!: "Since the prime factorization of 10! is $7 * 5^2 * 3^4 * 2^8$. Explain why there is only one 7 and two 5s, four 3s and eight 2s." This activity is not to compute 10! or even find the prime factorization of 10!. The students need to understand the concepts of prime factors and factorials in order to solve this problem. An example to use in an algebra class could be: "Compare and contrast this function with its parent function, $f(x) = -3x^2 - x + 2$". The students would first need to identify the parent function that matches the given function and know what the characteristics of the parent are, then the students would need to find comparable characteristics of the given function (axis of symmetry, vertex, direction of opening, and which is wider).

NCTM (2000) states that students need to be fluent in operating with algebraic expressions, solving complex equations and judging whether the answers are reasonable. Another important skill students acquire when they are confident of operating with algebraic expressions is the ability "to re-express functions in ways that reveal different types of information about them" (NCTM, 2000, p. 301). This understanding can help students with the next big concept: equations.

Equality

The equal sign (=) is a pivotal part of solving linear equations and another troubling part of linear equations for many students. Researchers have found that in elementary mathematics classes, most students use the equal sign as an operational sign to mean "give" or "make" (Behr, et al., 1980; Falkner, et al., 1999; Kieran, 1981; Kieran & Sfard, 1999; MacGregor & Stacy, 1997). Saenz-Ludlow & Walgmuth (1998) defined an operational understanding of equality as "a command to perform an arithmetical operation" (p 153). When many students have worked out problems like the following: 3 + 5 = 8 * 7 = 56 / 2 = 28, misunderstandings are bound to occur. Using the equal sign as a delimiter can often lead to a misunderstanding of equations. The transition from an operational understanding of equality to a relational understanding of equality is difficult.

Understanding Equality

For example, when presented with a task such as: 8 + 4 = □ + 5, a student limited to an operational understanding of equality would likely respond with 12 or 17 (Falkner, et al., 1999). Once the problem was modeled with manipulatives, students would then be

able to figure out the correct value, and thereby develop a more sophisticated understanding of equality. Successful algebra students tend to be those who have acquired a relational understanding of equality before taking algebra. Many researchers consider quantitative sameness as the relational understanding of equality (Behr, et al., 1980; Saenz-Ludlow & Walgamuth, 1998; Falkner et al., 1999; Kieran, 1981). Students with a relational understanding of equality are more apt to understand and use appropriately the properties of equality and not need to memorize all of the properties.

Kieran (1981) reports the results of a teaching experiment in which she was able to help students develop a relational view of equality. The following is a description of what occurred:

1) Students developed arithmetic identities (i.e., $6 + 9 = 6 + 6 + 3$)
2) Students justified why those were true
3) Students expanded the arithmetic identities to have many operations (i.e., $8 \bullet 2 - 5 = 5 \bullet 2 + 1$)
4) Students placed their finger over one of the numbers to see if other students could determine the missing number.
5) The teacher wrote down some of the arithmetic identities with boxes over a specific number.
6) Students replaced a number from the identities with a letter – if the number was repeated in the identity, the students replaced the repeated number with the same letter.

Kieran (1981) used the teaching experiment with six 12 – 14 year-old students.

By the end of the study, the students had a better understanding of relational equality as evidenced by the students' ability to write algebraic equations and explain that each equation meant one side "has the same value as" (p. 323) the other.

Mathematical Beliefs

Mathematical beliefs tend to be stronger than views about mathematics. Although, there is not common ground for the definition of mathematical beliefs among mathematics education researchers, many researchers use mathematical beliefs implicitly in their research (Törner, 2002, p. 75). Views about mathematics tend to be changeable during a student's lifetime. Carlson et al. (1999) theorized and researched students' mathematical views. A survey was developed and validated to show students' views about mathematics fall into six categories.

Students' views about mathematics

Changes in students' views about mathematics were assessed in this study. From Carlson et al's (1999) research, there are two overarching dimensions: pedagogical and epistemological. Each dimension has three themes. The three themes under the pedagogical dimension are: the "learnability" of mathematics, the use of critical thinking in mathematics, and personal relevance of mathematics. The three themes under the epistemological dimension are: structure of mathematics, methodology of mathematics, and validity of mathematical knowledge. Here are the operational definitions used for this study:

Students' views about the "learnability" of mathematics

Mathematics is learnable by anyone willing to make the effort rather than by a few talented people. Achievement depends more on persistent effort rather than on the influence of the teacher or textbook.

Students' views about the use of critical thinking in mathematics

For meaningful understanding of mathematics, one needs to:

Concentrate more on the systematic use of general thought processes rather than on memorizing isolated facts and algorithms.

Examine situations in many ways, and not feel intimidated by committing mistakes rather than follow a single approach from an authoritative source. Look for discrepancies in one's own knowledge instead of just accumulating new information.

Reconstruct new knowledge in one's own way instead of memorizing it as given.

Students' views about the personal relevance of mathematics

Mathematics and related technology are relevant to everyone's life rather than being of exclusive concern to mathematicians.

Students' views about the structure of mathematical knowledge

Mathematics is a coherent body of knowledge about relationships and patterns contrived by careful investigation rather than a collection of isolated facts and algorithms.

Students' views about the methodology of mathematics

The methods of mathematics are systematic and generic rather than idiosyncratic and situation specific. Mathematical modeling for problem solving involves more than selecting formulas for number crunching. Mathematicians use technology to enhance their ways of solving problems rather than to allow them to get quick, easy solutions.

Students' views about the validity of mathematical knowledge

Mathematical knowledge is validated by logical proofs rather than by correspondence to the real world (Carlson et al, 1999).

Technology helps students' understanding of algebra

Researchers have found that computers can remove the impetus of routine calculations from learners, allowing learners to critically examine the problem (Waits & Demana, 2000; Barton, 2000; State of the Art: Mathematics, 1993). Computers allow students to visualize mathematical concepts and understand the concept as studied from multiple perspectives, even in three-dimensional graphs and pictures. Most importantly, researchers have found that students' conceptual knowledge is enriched when working with technology, yet their procedural knowledge is not diminished (Cassity, 1997; Barton, 2000; Cave, 1995, Yerushalmy, 2004). This evidence suggests that, "when used appropriately these technologies do assist in increasing conceptual understanding without adversely affecting procedural knowledge" (Barton, 2000, p. 4). Appropriate use includes teaching and learning with an emphasis on problem solving, visualizing, and exploring

concepts.

Technology can help students to build conceptual knowledge. When used appropriately, using a computer can be a very effective way to ensure that students can build their own connections to the mathematics in the world around them. Technology can help students to see how mathematics has value (Yersuhalmy, 2004; Barton, 2000; Cassity, 1997) and the use of motivating problems can help students as well (Chazan & Yerushalmy, 2003; Heid et al., 1995).

Technology assists with motivation

Problem solving and the visualization of concepts are improved with the use of computer technology, but new instructional materials are needed to utilize these novel tools in classrooms (Barton, 2000; Waits & Demana, 2000; Graham & Thomas, 2000). What is used and how it is used in the classroom are important considerations. ASK ME-Algebra's use of technology can be another motivating influence for students.

As an example, the aim of the ASK ME-Algebra curriculum is to present mathematical concepts within real-world situations in order to enhance the application of mathematics in everyday life. Topics include:

- Eliminating speeding through neighborhoods
- Fundraising issues for an animal shelter
- Increasing recycling and reducing waste

Students can interact with a dump truck to draw a graph of time versus distance as an example of Flash animations used with ASK ME-Algebra. The animation is similar to the way graphing calculators and motion detectors can be used to discover the

relationship between distance from the motion detector and time of travel. This discovery leads to an important concept in algebra: distance varies directly with time.

Research has shown if students explore the concept of distance varies directly with time using the multiple representations of functions to connect variables and dependence relationships then students gain understandings. (Heid & Edwards, 2001; Brenner et al., 1997).

Research shows that when students are presented with realistic problems they can be more motivated to solve them (Malloy & Malloy, 1998; Cobb et al., 1992; Kilpatrick, 1987). Students often see a real-world situated problem as something worth the effort; as they work through the problem, they learn about mathematics at the same time (Boaler, 1997; Lehrer, Schauble, Strom, and Pligge 2001, Maehr and Anderman, 1993). "Learning-Appropriate goals" (Barron, Schwartz, Vye, Moore, Petrosino, Zech, Bransford, and CTGV, 1998, p. 273) are also a factor in students' motivation. For example, Barron *et al* (1998) had students in the first year of the project launch rockets, but during the second year used a letter from NASA asking the students to determine which type of rocket goes higher. The students then learned to keep track of data, to measure the height of the flying rocket and to draw conclusions from their data.

The Jasper project demonstrates scaffolding students' learning with technology (Barron et al, 1998). "The Adventures of Jasper Woodbury" is a mathematics problem solving series that was developed at Vanderbilt's Learning Technology Center. One of the episodes in the Jasper series involves saving an injured eagle found by a hiker. The

students were highly motivated to save the eagle and to use the computer resources to help find the needed missing information to rescue the injured eagle.

Understanding Algebra

The ASK ME–Algebra course used the *Understanding by Design* (Wiggins & McTighe, 1998) WHERE[10] model to assess students' learning. The unit problem stated at the beginning of each unit sets up the anchor video, which introduces the problem in an engaging manner. The guided practice problems allow the students to explore concepts and solve the unit problem. The tutorials inside each "guided practice" activity also allow the student to become equipped with knowledge of a new concept. The "enduring understanding" questions in the graded assignments help students to reflect. Open-ended projects are the culmination of each unit, and the projects allow students to exhibit their solutions creatively. There are transfer questions in the graded assignments as well; for example, in Unit 2 the overarching or unit question pertains to the student helping the community to decide on a trash collecting pricing structure. There are questions about buying worms, recycling yard waste, and trash trucks traveling in front of a motion detector. Determining if students improve or elaborate on their responses to the transfer questions in the graded assignments over time is a way to confirm that the ASK ME-Algebra course is improving students' understanding of algebraic concepts. To determine improvement, students' responses would garner higher points when graded by the instructor using the grading rubric.

[10] WHERE is an acronym for Where are we headed?, Hook the student with engaging and provocative entry points, Explore and enable/equip, Reflect and rethink, Exhibit and evaluate (p.115).

How people learn (HPL)

Learner-centered, knowledge-centered, assessment-centered, and community-centered are the four concentrations of the (HPL) framework. Each area was used to create the Cartwaller community used in ASK ME-Algebra. The framework developed by Bransford, Brown, and Cocking (2000) has shown to help students understand mathematics and science.

Bransford, Brown and Cocking's (2000) framework emphasized learner-centered instruction; this was a guiding factor in the development of ASK ME-Algebra. Many teachers still work within the "sage on the stage" framework, but with ASK ME-Algebra the instructor is the "guide on the side". The activities and tutorials are scaffolded so that the student can work through the problems at their own pace and use all of the program's resources, if needed. In this way, the students are placed in charge of their own learning environment and are expected to make sense of activities (Battista, 1999; Malloy & Malloy, 1998; Chazan & Yerushalmy, 2003).

A knowledge-centered environment includes the following:

- An understanding of the concepts and transfer of that knowledge
- An emphasis on sense-making
- An integrated understanding of the concepts

ASK ME-Algebra is knowledge-centered, since the curriculum revolves around the functions-based approach to algebra. With the function as the central focus of the curriculum, most students can make sense of the material. Real-world situations are posed to the students, and help students use the concepts they are investigating. The

conceptual knowledge students learn while solving the real-world problems can be integrated into their mental concept maps (Bransford, Brown and Cocking, 2000).

Assessment-centeredness was very important to the development of ASK ME-Algebra. The ASK ME assessment methodology differs from traditional curriculum in these respects: (a) opportunities for feedback on graded work by the instructor (b) opportunities for students to revise graded work. ASK ME-Algebra assesses each student's understanding to make certain that students can explain each concept in their own words. For example, students must describe elements of graphs and linear functions in Unit 1. Open-ended problems and transfer problems[11] are also a part of ASK ME-Algebra's assessment-centered environment.

ASK ME-Algebra uses community-centered activities to help students to learn that algebraic concepts are used outside of the classroom. In the first unit, the students develop a speeding fine for a neighborhood; in the second unit, the students develop a trash collecting fee structure; and in the third unit, the students develop fund-raising ideas for an animal shelter. The community members of ASK ME-Algebra pose unit problems to the students through the opening animation in each unit. The characters also help students to feel like part of a community of learners. One character tends to asks probing questions, one character tends to uses the technology correctly and one character tends to communicates mathematically.

[11] Transfer problems are similar in to the studied problem but different enough to cause the students to use methods recently learned. For example, linear equations are studied in a context of a community garden; a transfer problem deals with buying worms or even composting materials.

Conclusion

Research shows that students tend to struggle with variables, expressions, and equality when learning algebra, and that a new method of algebra instruction is needed to help students to overcome those struggles. Students should learn to master the many ways in which variables are used in mathematics. When students can determine expressions are equivalent then students are gathering the knowledge needed to succeed in algebra. A rational view of equality is a necessary concept for all students to grasp in order to be successful in high school algebra and beyond.

High school students have views about mathematics that are in flux. With positive experiences in mathematics classes, students' views about mathematics can be affected in a positive way. Over time, students should see that mathematics is useful and does not need to be avoided.

Chapter 3 Methodology

This study[12] explores two possible areas of impact of the ASK ME-Algebra curriculum:

- In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' understanding of concepts, such as variables, equality, and equivalent expressions?
- In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics?

The focus of this study is to examine the change in the treatment group's understanding of the concepts of variables, expressions and equality. This study also examines the change in students' views about mathematics after exposure to an online comprehensive and all-encompassing curriculum for Algebra I in an unfamiliar environment; an environment that is unfamiliar to low SES, rural high school students. A comparative component determines if the treatment group developed the same content knowledge as the control group (Romberg, 2003). The quantitative data consists of

[12] This study has IRB approval, see Appendix G. The consent forms appear in Appendix A.

responses to Texas Assessment of Knowledge and Skills (TAKS) released test items[13]. The methodology, explained in this chapter, is the design experiment.

The expectations of the results of this study are that the treatment group will significantly improve their understanding of variables, expressions and equality; the treatment group will understand as much as or more than the control group about variables, expressions and equality; and the treatment groups' views about mathematics will improve (i.e. change from folk to mixed or expert).

Research Design

The following is a diagram of the experimental research design used for this study:

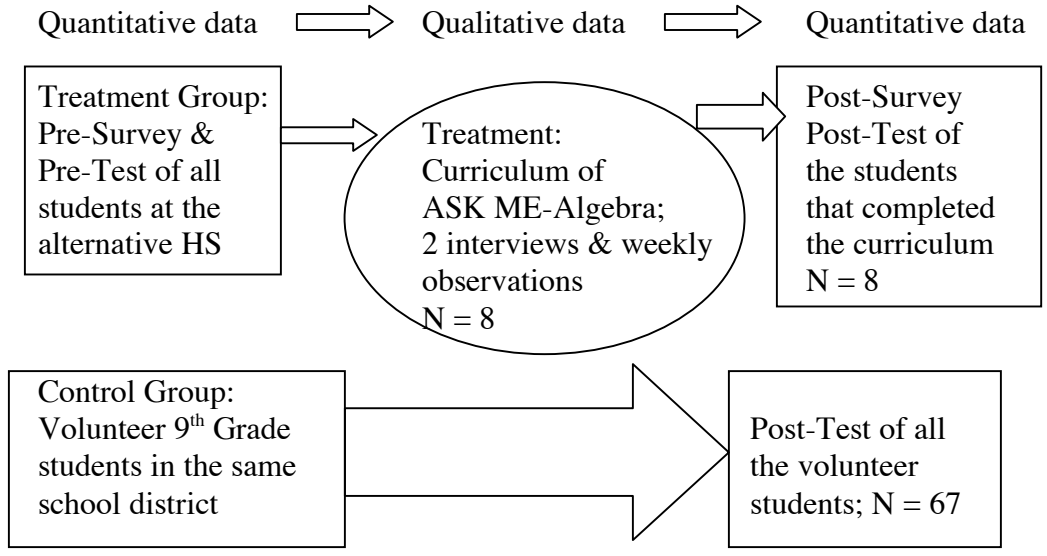

Figure 1: Experimental Research Design (Miles & Huberman, 1994)

[13] See Appendix N for the copyright permission email from TEA.

Since the treatment group has experienced algebra class at least once, sometimes twice, a pre-test, administered to determine how much algebra content knowledge students had acquired, will be conducted before the students start the ASK ME-Algebra curriculum. The control group, for the most part, have not experienced algebra previously, so it is assumed that they would have little to no algebra knowledge and a pre-test will not be administered to them.

The treatment group will also answer a survey about their views of mathematics before the treatment. Two interviews will be conducted with each student in the treatment group. The first interview will be used to validate their survey responses and it will be conducted during the first three weeks of treatment. The second interview will be used to determine if any change occurred in their views of mathematics over the nine months of curriculum use and the second interview will included tasks for the students to work in order to determine if their knowledge of the three algebraic concepts was strong. The post-test will be used with the treatment group and the control group to compare each group's scores. The control group is considered the normalized group and if the treatment group can score as high or higher than the control group after treatment then it will be possible to conclude that the treatment group improved their knowledge about variables, expressions, and equality after experiencing the ASK ME-Algebra curriculum.

During the treatment, the students will save their work online to their Guided Practice problems, Activities, Graded Assignment problems, their pre-& post-tests, and pre-& post-surveys. Observations of the classroom, the setting, the technology

challenges, and collection of the teacher's corrections of the curriculum will also occur during treatment.

Setting

The treatment group is from Gaggle High School and Gaggle High School was one of the sites for the initial implementation[14] of the ASK ME-Algebra course during the 2004-2005 school year. The principal and mathematics teacher agreed to allow all of their students to use the ASK ME-Algebra curriculum and receive credit for high school algebra.

Gaggle High School is an alternative high school for students served by Goose ISD, a low SES, rural Central Texas school district. Students voluntarily apply to attend Gaggle High School for many reasons, but the main reason is to be able to work full-time and still graduate from high school, preferably with their cohort. Students are able to attend school and work at full-time jobs simultaneously because Gaggle High School holds classes for four hours each day. The students are self-motivated: they seem to realize that Gaggle High School is their last opportunity to receive a high school diploma. After working at Gaggle High School one summer, I realized that all of the students I worked with were highly motivated to graduate from high school, even though the students had a variety of reasons for coming back to school to earn a diploma (three of the students were out of school for an extended period of time).

[14] There were five school districts and fourteen teachers participating in the initial implementation. Gaggle High school was recruited to participate in the initial implementation since the teacher and principal were willing to allow the researcher access to their students.

The school structure allows students to work on each high school course at their own pace; for example, they can earn one-half credit in any course in six weeks[15]. The atmosphere of the school was inviting and supportive. Students are accountable for attending classes daily and earning credits in a timely manner.

The mathematics teacher, Pamela C., is enthusiastic about using the ASK ME-Algebra curriculum with her students. Since taking courses towards her Master's degree in Education, she was intrigued by the idea of teaching algebra using a function-based approach but had not found any support materials to do so. Therefore, she is willing to use all facets of the ASK ME-Algebra course[16] with her algebra students. She conducts four 60-minute classes at Gaggle high school, each of which consists of 16 – 20 students enrolled in either Algebra 1A, Geometry, Mathematical Modeling with Applications, or Algebra II.

Pamela's classroom has five computers; four are for student use. In each class period, 3 – 4 students are enrolled in Algebra I. Each student has access to a computer for the duration of the school year. The computers are against one wall of the classroom. The other students (not enrolled in Algebra I) are at desks in the center of the classroom. There are graphing calculators in the classroom that each student has access to; the calculators are stored on the teacher's desk.

[15] One semester is equivalent to one-half credit.

[16] I conducted a two-hour one-on-one training with Pamela on how to use ASK ME-Algebra, grade assignments, send messages, and make announcements. I was there weekly to assist with any technological problems.

There are three male students and five female students. Four of the eight students live outside the city limits of Goose. The county that Goose ISD is located in, is considered a low SES area.

Participants

The students involved in this study are from the same school district. Both groups are enrolled in Algebra 1, which is a requirement to graduate from high school in Texas. The treatment group consists of all of the algebra students enrolled at Gaggle High School and the control group consists of volunteers from the Ninth Grade Center. The following sections describe the students in detail.

Eight Case studies

Ana M. is a seventeen-year-old Latina sophomore who had attended a high school Algebra 1 class for three years. She also attended a Geometry class during her tenth-grade year but she did not receive credit for either class. She moved from a large urban school district in central Texas to Goose ISD during fifth grade.

Bubba B. is a sixteen-year-old Anglo sophomore, who has been through high school Algebra 1 once in a non-accredited private religious school. Bubba has only attended schools in Goose ISD.

Barbara W. is a sixteen-year-old Anglo sophomore who has been through Algebra 1 once. She admits that during the second half of her freshman year she had poor attendance. Barbara attended 8th grade in Dallas, but has attended Goose ISD for all other grades.

Günter G. is a seventeen year-old Latino sophomore who previously attended one year of Algebra 1. Günter has only attended schools in Goose ISD.

Jean C. is a seventeen year-old Anglo sophomore. She was in algebra for one year during the ninth grade. Jean has only attended schools in Goose ISD.

Mary G. is a seventeen year-old Anglo sophomore. She spent three years in Algebra 1. She made low Cs in all of her other math classes. Mary has only attended schools in Goose ISD.

Nan M. is a seventeen year-old Anglo sophomore. She attended one algebra class before attending Gaggle HS. Nan has only attended schools in Goose ISD.

Vince P. is an eighteen year-old Latino sophomore. He was in algebra classes during two different years. Vince did not attend during the 2003-2004 school year, before being admitted to Gaggle HS for the 2004-2005 school year. Vince has attended only Goose ISD schools.

Comparative Study

Goose ISD's Ninth Grade Center agreed for their Algebra 1 students (N=67 for the control group) to volunteer to take the algebra test. The students at the Ninth Grade Center are either freshmen or second year freshmen[17]. The Ninth Grade Center uses a textbook as their main source for algebra curriculum, along with a majority of the schools in the state. The textbook uses an algorithmic approach to algebra. The teachers and students use TI-83 calculators in the classrooms. The three algebra teachers agreed to

[17] In order to enroll at the high school campus, a student must have six credits (one credit equals two semesters) from the Ninth Grade Campus.

give the 19-question test from TAKS released items to their students during the last two weeks of February 2005. This time frame was chosen to insure that the 9th grade students had covered the same material as each Gaggle HS student had through the ASK ME-Algebra course. Generally, the first chapter in many state-adopted algebra textbooks places an emphasis on order of operations and evaluating expressions. Those subjects were embedded in ASK ME-Algebra 1A and did not have a separate unit covering that material.

The data from the Ninth Grade Center will be used to compare the treatment group's algebra knowledge to the algebra knowledge that freshman algebra students are expected to gain in their first semester of learning algebra. The expected results are that the treatment group learns the same amount of content or more than the control group.

Control Group

The teachers will collect the consent forms[18] first and then allow the students to take the 19-items from released TAKS tests[19] during class. The post-test includes questions about demographics: age, gender, overall grades, highest mathematics class taken, and zip code but names are not included on the test. Six of the 67 students are 16 year-olds (ninth grade students are 14 to 15 years-old). Thirty-six of the 67 students in the control group were female; the other 31 students were male. Eight of the 67 students reported receiving failing grades in mathematics classes. Everyone in the control group

[18] See Appendix A for a copy of the consent forms.
[19] I volunteered to analyze the results and share the analysis with the principal.

reported not taken any class higher than Algebra 1. Not all of the students in each teacher's 4 – 6 classes turned in consent forms to be part of this study.

The ratio of boys to girls is similar to that of the treatment group: 31 to 36. The demographic data suggests that the control group lives in the same areas of the county as the treatment group: 18 out of 66 (one student left this question blank) reported living outside of the city of Goose.

The control group is at the Ninth Grade Center in Goose ISD and is on a 7-period bell schedule, where classes are 50 minutes long. Each student either passed 8th grade or are returning 9th grade students. The environment of the school is structured and caring. The principal of the Ninth Grade Center, Mr. Harmon, agreed to allow his three algebra teachers to administer the algebra test. In order for the students to attend Goose High School, they have to pass at least 5 of the 7 classes they are currently enrolled in.

The three algebra teachers at the Ninth Grade Center are Connie C., the department chair, who has taught mathematics for 10 years; Richard P., who is a second-year teacher and coaches basketball; and Monty T., who is a first year teacher and coaches baseball. The teachers use a state-adopted textbook (Prentice Hall) as their primary source of curriculum. Each teacher's classroom has a classroom set of graphing calculators and an overhead screen for the teacher's calculator. Each classroom has one computer for the teacher's use.

The pre-test, that was administered to the treatment group, shows that treatment group's knowledge of algebraic concepts are similar to what was assumed about the control group: very little algebra knowledge (correct responses out of 19: $\bar{x}$ = 5.5 and

median is 5.5). Since the students in both groups live in the same community and have similar knowledge of algebra, I contend that the two groups are similar enough to compare their algebraic content knowledge with a only a post-test of algebraic content knowledge from the treatment group and a test from the control group after they covered similar content.

Data Collection

The pre-test from released TAKS tests items is used to determine if the treatment group has any algebraic concept knowledge before using the curriculum. It is assumed that the control group would have none to very little algebraic concept knowledge, so the pre-test will not be administered to the control group. The post-test consist of the same 19-questions as the pre-test and will be used to determine if a change occurred in the treatment group's understanding of three algebraic concepts: equality, expressions and variables. The treatment groups' knowledge will be compared to control groups' knowledge of the same subject matter using the raw scores of the algebra test. My hypothesis is that there would be a significant change in the treatment groups' post-test scores and that the treatment group would understand as much as or more than the control group of the algebraic concepts: variables, expressions and equality.

Data from the test to be collected from the control group, will be used to compare the control group's knowledge of algebraic concepts to the treatment group's knowledge of algebraic concepts after learning the same concepts and determine if the two groups are similar. Since the main controversy with PBI is that students do not learn basic skills, this comparison will be used to show that the treatment group learned at least what it

should have learned – or more. The 19-question test of TAKS released items will be administered to the control group in late February 2005 in order to ensure that the control group has covered the same course material as the treatment group in one semester of ASK ME-Algebra. The control group, using a state-adopted textbook as their primary source of curriculum, should cover the algebraic concepts approximately six weeks after the beginning of school.

The interviews will focus on all of the students in the treatment group. The all of the students will be interviewed twice while working through their first semester of the ASK ME-Algebra curriculum. The first interview will determine the treatment group's experience with mathematics classes, and will be used to determine their current views about mathematics. During the second interview, the students will perform structured tasks involving variables, expressions and equality. My hypothesis is that the students would be able to perform all of the tasks correctly.

The treatment group members will also complete the VAMS survey about their views of mathematics before they start the course and they will repeat the same survey at the end of the treatment. The survey asks questions regarding the students' views about mathematics. My hypothesis is that the students' views would improve their views about mathematics (i.e. change from folk to mixed or expert) after taking the ASK ME-Algebra course.

There are 12 students enrolled in the ASK ME-Algebra 1A course. All of the students will take the pre-test of TAKS released algebra items and the pre-survey, and answer the questions in their initial interviews. I will be observing the classroom weekly

at various times during the week to conduct interviews and record students' interactions with the course and technical issues either the teacher or the students' encounter.

The follow-up interview will be conducted in March and April 2005. Every student still enrolled in Gaggle HS will be interviewed at that time. Students will complete the post-survey as they finish Unit 3. All of the work students save online will also be collected for this study. The collected work will include their responses to the Guided Practice, their responses to the Graded Assignments, and the amount of time each student spent on each activity, including tutorials.

Methodology

Since the basis of the curriculum for ASK ME-Algebra is constructivist, it is only natural to use a research design for this study that complements that theoretical framework. Design experiments consist of researchers designing the curriculum, doing progressive refinement of the curriculum, implementing the curriculum in the learning environment, observing the different affects of the treatment, characterizing the complex social situation, collecting large amounts of data, and analyzing many different aspects of the design.

Instrumentation and Measures

The instruments to be used in this study are a survey, interview questions, and 19-questions from released TAKS tests. The following sections will discuss each instrument in detail.

Survey

A pre- and post-survey will be administered to the treatment group to determine if there is a change in students' views about mathematics after treatment. The survey to be implemented is *Views About Mathematics Survey*[20] (VAMS) (Carlson, 1997a), which was developed for use with college students. The survey has been used with high school students (A. Bloom, personal communication, July 24, 2004). To develop the survey, many iterations of writing questions from research, administering the questions, and conducting follow-up interviews were conducted. The survey was also given to mathematicians and their responses were categorized as expert responses. College mathematics students' (lower class and upper class) views were compared with the experts' views about mathematics. A student scored low, a folk view, if their view was very different from the experts' views. A mixed score resulted if a student's view was slightly different from the experts' views. Expert scores resulted when students' views were similar to the mathematician's views. The VAMS measures students' views towards mathematics.

Seven questions have shown the most change after treatment (see Table 1). I anticipated that this survey would help to determine if the ASK ME-Algebra course changes students' views towards mathematics after treatment (Carlson, 1997a); this relates to my second research question.

Table 1 presents the experts' views, the mixed views and the novices' views on seven specified questions that have shown to have the most significant shifts in

[20] See Appendix B for the full questionnaire.

participants over time in studies conducted by Carlson (1997a). Students in Carlson's study had the following example choices to select from, "If I had a choice: (a) I would never take any mathematics course (b) I would still take mathematics for my own benefit". Students in Carlson's study moved from "mostly (a)", never take any mathematics course, to "mostly (b)", still take mathematics for my own benefit, after the intervention.

The shifts expected in the treatment students' views about mathematics are in the questions from Table 1. Interview data analysis can help to triangulate these conclusions.

The survey questions that measure a change in "learnablity of mathematics" are 5, 9, 11, 13, 19, and 34. The survey questions that measure a change in "personal relevance of mathematics" are 6, 7, 20, 27, 33, and 36. The survey questions that measure a change in "the role of reflective thinking in mathematics" are 10, 17, 18, 24, and 42. The survey questions that measure a change in "structure of knowledge in mathematics" are 14, 25, 30, 32, 35, 39, and 40. The survey questions that measure a change in "methods in mathematics" are 12, 16, 23, 29, 37, 38, and 41. The survey questions that measure a change in "the validity of mathematical knowledge" are 8, 15, 22, 26, 28, and 31.

Table 1: VAMS questions that has shown movement, adapted from Carlson (1997a)

Questions which showed students' views shifted over time	Experts' Response	Mixed Response	Novice Response
6. If I had a choice: (a) I would never take any mathematics course. (b) I would still take mathematics for my own benefit.	Only b; Mostly b rarely a	Equally a & b; More b than a	More a than b; Mostly a rarely b; Only a
13. In mathematics, it is important for me to: (a) memorize technical terms and mathematical formulas. (b) learn ways to organize information and use it.	Equally a & b; More b than a; Mostly b rarely a; Only b	More a than b	Only a; Mostly a rarely b
15. After I go through a mathematics text or course materials and feel that I understand them: (a) I can solve related problems on my own. (b) I have difficulty solving related problems.	Only a; Mostly a rarely b	More a than b	Equally a & b; More b than a; Mostly b rarely a; Only b
19. How well I do on mathematics exams depends on how well I can: (a) recall material in the way it was presented in class. (b) do tasks that are somewhat different from the ones I have seen before.	Mostly b rarely a; Only b	More a than b; Equally a & b; More b than a	Only a; Mostly a rarely b
23. Collecting and graphing real world data is useful for: (a) determining patterns and making general predictions. (b) obtaining numerical answers to specific problems.	Only a; Mostly a rarely b; More a than b	Equally a & b; More b than a	Mostly b rarely a; Only b
25. When solving a challenging mathematics problem, a mathematician: (a) makes many incorrect attempts. (b) moves directly to a correct solution.	Only a; Mostly a rarely b	More a than b	Equally a & b; More b than a; Mostly b rarely a; Only b
41. Scientists use mathematics as: (a) a tool for analyzing and communicating their ideas. (b) a source of factual knowledge about the natural world.	Only a; Mostly a rarely b; More a than b	Equally a & b	More b than a; Mostly b rarely a; Only b

Interviews

The two interviews will be performed to determine: a) students' views of mathematics and b) students' understanding of variable, expression, and equality. Audiotaped interviews will be conducted following the completion of Unit 1 and at the completion of Unit 3 (which is the end of the treatment), resulting in two interviews with each of the students in the treatment group.

Each interview will take place with a tape recorder and digital recorder. The interviews will be about fifteen to twenty minutes in length. I will go to Gaggle High School to conduct the interviews in the commons area.

The first interview[21] will focus on asking the students questions about their personal lives, in order to develop a rapport with the student (Seidman, 1998). For example, "What are your hobbies?", "What are your plans after you graduate from high school?" Their responses to the algebra-related questions (e.g. "What is your definition of a function?") will be used to generate personalized questions to be used in final interview. The students will be given an opportunity to ask questions about the course or my research.

The second interview, conducted as the student finished Unit 3, will occur approximately 5 months after the first interview. The final interview focuses on tasks used to determine student's understanding of variables, expressions and equality. The tasks, found in research, will be given to the students, and their thoughts about how they worked the tasks will be elicited.

Major benchmarks in student understanding of algebra are the use of variables, the use of expressions and a relational understanding of equality (Behr et al., 1980; Brenner et al., 1997; Graham & Thomas, 2000; Kieran, 1981; Kieran & Sfard, 1999; Usiskin, 1989). An example of a question that can elicit the understanding of equivalent expressions is the following: "Is $x - y = z - y$ true always, never, or sometimes? If sometimes, when?". A question about students' understanding of variables is, "Simplify

3a – (b + a)" (Graham & Thomas, 2000). An example of a question that can elicit students' understanding of equality is, "What number should be in the blank: 2 + 17 = __ + 13?" (Kieran & Sfard, 1999).

[21] The interview protocols are in Appendix D and Appendix E.

***TAKS Released Algebra Items*[22]**

Table 2: Categories of Concepts relating to TAKS released items

Concept	**Items from Pre-Test**
Variables	2, 12, 20, 22, and 35
Expressions	10, 23, 24, 25, 26, 27, 32, and 34
Equality	4, 8, 9, 29, 31, and 36

Key: TAKS Obj. 1, Obj. 2, Obj. 3, Obj. 4, Obj. 10

Using the Texas Assessment of Knowledge and Skills[23] test as a guide, 19 questions were founded to address the acquisition of the concepts of variables, expressions and equality. The versions of TAKS used were Spring 2003, Grades 8 and 9, and Spring 2004, Grades 7, 8, 9, and 10. Of the 19 questions, 17 were answered by all Texas students at a rate of 50% or higher. The pre- and post-test contained the same questions. For the treatment group, the pre-test is administered online and all at once. The post-test questions will be given to the treatment group as Graded Assessment problems online and throughout the course (this is beyond my control). The control group will receive the 19 post-test questions on paper (this is also beyond my control).

The TAKS tests measure students' achievement throughout their school career (grades 3 – 11). The questions gathered for the use of determining the treatment groups' increase in knowledge of algebraic concepts only used questions aligned with the algebra

[22] The survey questions are in Appendix B.

objectives of TAKS. I contend that this use was not beyond the original purpose of the TAKS test.

The following are examples of questions that address each concept:

[23] A statewide assessment program, the Texas Assessment of Knowledge and Skills (TAKS™) was implemented beginning in spring 2003 and replaced the Texas Assessment of Academic Skills (TAAS).

Variables

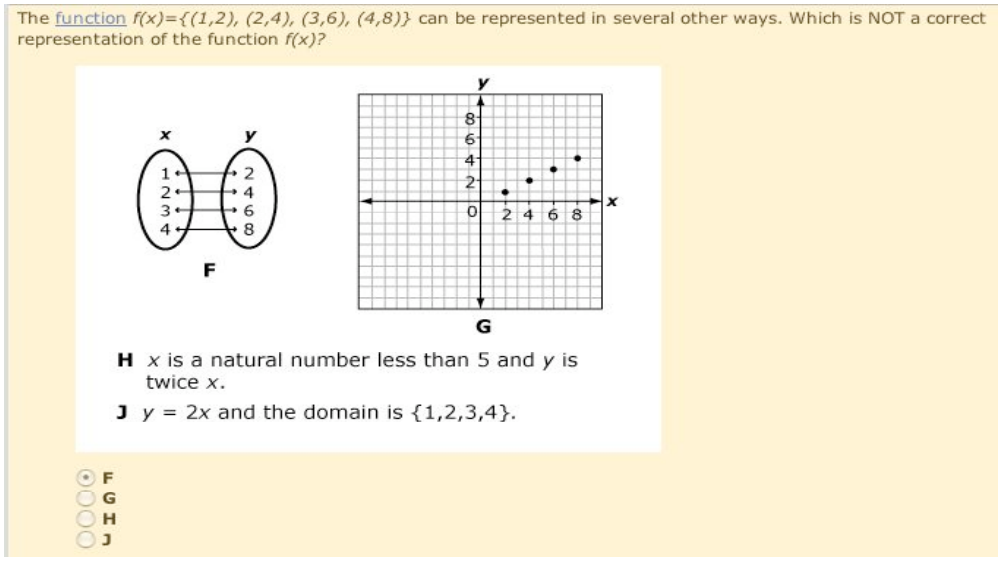

Figure 2: Question 2 of the Pre-test.

This question elicits understanding of variables since the student realizes that x and y have many values and uses the values of x to find the values of the variable y. The questions in the pre-test that address the concept of variables are 2, 12, 20, 22, and 35[24].

Expressions

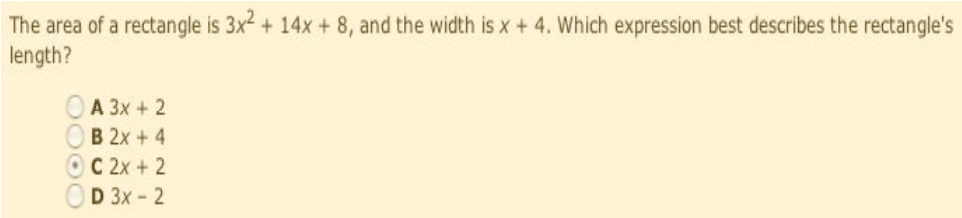

Figure 3: Question 25 from the Pre-test.

[24] See Appendix C for all of the questions.

This question elicits an understanding of expressions by having the student work with an expression for area of the rectangle and the width of the rectangle to find the expression to represent the length of the rectangle. The questions that address the concept of expressions are 10, 23, 24, 25, 26, 27, 32, and 34[25].

[25] See Appendix C for all of the questions.

Equality

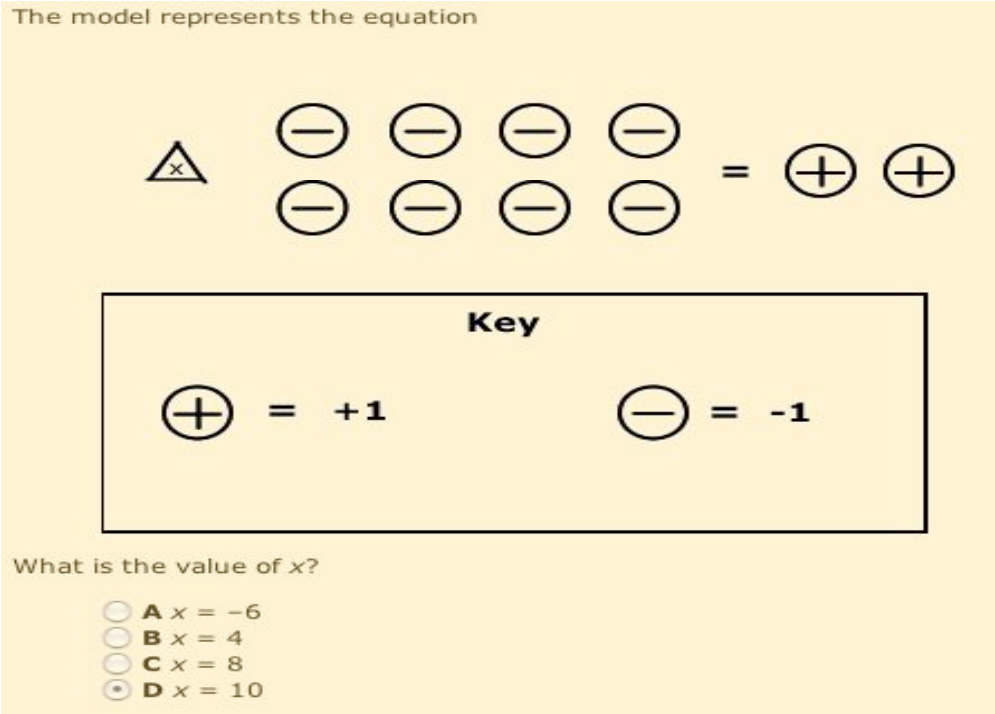

Figure 4: Question 31 from the Pre-test.

This question elicits an understanding of equality since the student could change the model into an equation and solve the equation using their understanding of equality. The questions that address equality are 4, 8, 9, 29, 31, and 36[26].

The released TAKS algebra items[27], embedded throughout the ASK ME-Algebra course in the graded assignments, are used to determine if students not only understand the concepts of variables, expressions and equality but also the concepts taught in each unit of the ASK ME-Algebra course.

[26] See Appendix C for all of the questions.

Data Analysis

The survey data will be changed from values of 1 through 7 to categories of Folk, Mixed or Expert. This translation does lose information since a student might have chosen the number 4, which is considered Mixed when he or she meant to have chosen the number 5, which is considered Folk.

The test data is collected as alphabetical choices of a – j and translated into a correct or incorrect choice. Answer choices are given by the test writers to distract the students but partial credit is not awarded.

The interview data are transcribed and themes are chosen from the answers of the students. The context of the question and answer might be separated in analysis.

The students' time on task will be organized by activity to find any general patterns. The students' work from the Graded Assignments will be examined if there is a conflict between the interview tasks and the post-test scores.

Quantitative Analysis

The data will be analyzed as follows: Two Sample T-test ($\alpha < 0.01$) will be run between post-test data of the treatment group and the test data of the control group to determine if the treatment group grasped the algebra concepts as much as or more than the control group. Since the control group's size is greater than or equal to 40, using a t-test will be valid.

Paired Sample T-test ($\alpha < 0.01$) will be used with the pre- and post-test data of the treatment group to determine if the treatment had an effect on the learning of

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[27] As part of my job with the OMI, my job was to find the TEKS objectives that aligned with each unit and to decide which TAKS items to use in each Graded Assignment. There are 38 TAKS released items in the

algebraic concepts by the treatment group. Since the treatment group size is less than 15 then the data will be checked for outliers so that using a t-test will be valid.

The treatment groups' responses to the 42 Graded Assignment problems within the ASK ME-Algebra course will also be used to determine what concepts the treatment group learned. The interviews, Graded Assignments, and post-test responses will be used to help triangulate the results of the quantitative data.

The raw scores of pre- and post-tests of the treatment group will be used, since I am not interested in comparing each student to the mean of the class. I am interested in seeing a change of their scores from before treatment and after treatment. I am also interested in seeing if the treatment group's scores are similar to the control group's scores.

Validity of the Study

After determining if the treatment group's algebraic content knowledge increased or decreased by using the pre- and post-test scores, then the interview tasks will be examined to see if each student was successful or not at completing the tasks. Another data point that will be examined is the student's Graded Assignment responses. A majority of the responses in the ASK ME-Algebra course are written, so if a student has a solid understanding of the concepts of variables, expressions, and equality, then that understanding should be very apparent in their written responses to the Graded Assignment problems. The observation notes can also be a source of what occurred in the classroom.

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course but only 19 of them deal with equality, variables, or expressions.

This is a study of 8 students, so generalizations about the population of Algebra 1 students in Texas are not possible.

Contributions and Limitations of the Study

Mathematics educators and researchers interested in a Functions-Based Approach (FBA) to algebra would be one audience that this study will be of interest. This study could also appeal to people in the mathematics community interested in Problem-Based Instruction (PBI).

A broader audience would be mathematics educators who want to find a better method of algebra instruction for all students. The curriculum of the ASK ME-Algebra course is challenging and accessible to most of the high school students in Texas that have yet to receive credit for Algebra 1.

Since the number of students in this study is small, inferences would not be recommended. The case studies will allow researchers and others to learn about the use of the curriculum with students as independent learners.

Chapter 4 Results and Analysis

This chapter contains the data collected from interactions that Gaggle HS students had with the first semester of the ASK ME-Algebra course. The following data were collected from each of eight Gaggle HS students: (a) pre- and post-tests of released TAKS items[28]; (b) pre- and post-VAMS[29] surveys; (c) student's work, both activities and Graded Assignments; (d) two interviews, once after he or she finished Unit 1 to determine their math background, and again after he or she finished Unit 3 (the end of the first semester of ASK ME-Algebra); and (e) weekly observations of the classroom.

There were sixteen graded assignments in the three units of the first semester of ASK ME-Algebra. Each graded assignment contained around 10 questions; most of the questions were open-response but a few were multiple-choice problems (i.e. TAKS test items). Within the graded assignments, the teacher of the treatment group scored the open-response questions and the system automatically scored the multiple-choice questions. The teacher had the option to request that students redo any of the questions, including the multiple-choice questions.

ASK ME-Algebra Overview

Unit 1 includes an introduction to functions in which students learned the vocabulary words: domain, range, independent, and dependent. The multiple representations of a function are also covered: as a table of values, as a graph, verbally and algebraically. Unit 2 discusses linear functions in which students learn about rate of

[28] The items are in Appendix C.

change, slope, y-intercept, the linear parent function, and how to write algebraic rules when given two points (either in table form or on a graph). In Unit 3, students learn that equations are specific instances of functions. The students are introduced to methods of solving equations using approaches other than symbolic: using graphs, using tables, and using "guess and check." The subject of inequality and solving inequalities are covered in Unit 3, as well.

The Graded Assignments (GA) comprise of student work that is graded by a teacher. The teacher can give each student the grade he or she earned or request that the student redo a question and give the student helpful feedback on what concept he or she missed and where to find help on that concept in the activities. The concept of variables is assessed in Unit 1 GA 1, 2, 3, and 4, and in Unit 2 GA 2, 4, and 5. The concept of equivalent expressions is assessed in Unit 2 GA 2, 4, and 5, and in Unit 3 GA 2. The concept of equality is assessed in Unit 3 GA 1, 2, and 3.

Activities and Tutorials

The work saved by each student while interacting with the first semester of ASK ME-Algebra is recorded in the system. Unit 1 has 11 activities, in which each activity includes review exercises, tutorials, guided practice questions (that can be saved), and extra practice problems. Unit 2 has 13 activities and Unit 3 has 15 activities.

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[29] Views About Mathematics Survey

The tutorials[30] teach the main concepts. The guided practice problems lead the student through thought processes to gain understanding of the concepts and prepare the student for the Graded Assignment questions. Most of the activities have a tutorial or two the students should interact with and view. Each Graded Assignment has three to four activities that lead up to it.

One way to determine if the students were interacting with the full curriculum is to look at their saved work. The data generated by the students in the treatment group showed that saving their work was not done after October 2004. There are three possibilities for the lack of saved work by the student at Gaggle HS. The first is that the students did not work through any of the activities that were not saved by them. The second is that the students stopped saving their work after a certain point – maybe realizing that it was not part of their grade but still worked through the activities and viewed the tutorials. The final possibility is that the teacher was instructing every class[31] through each activity. The second possibility seems unlikely since many of the students during the first interview did not recognize any of the tutorials when I asked the students about specific tutorials. The last possibility seems likely since I did observe an occurrence of this happening. I did observe the teacher going over a question in the Graded Assignment that some of students were having trouble answering. From an

[30] For a complete listing of the tutorials by Unit and to examine the tutorials go to: http://www.dec.utexas.edu/askme/ALG1A.html

[31] There were four classes a day with four different subjects in each (Algebra I, Geometry, Algebra II and Mathematical Modeling with Applications).

observation on 10/19/2004, Pamela stated that she would be helping the students for 30 minutes twice per week after the TAKS test that would be given the following week.

After a conversation with Pamela on March 24, 2006, she verified that the students would copy everything (even misspellings and typos) as the class worked together to solve problems on various Graded Assignments. Pamela did state that she witnessed most of the students viewing the tutorials even though the students may not have worked the guided practice problems and did not save their work.

A culture of practice[32] at Gaggle HS is for students to take only the tests at the end of chapters and not do any of the work in the sections of the chapters. This accepted way of working affects the results of this study and this will be reflected on in the analysis of the next chapter. The students do usually not do any work that is not given a grade, and as long as the students make an 85%[33] or better on the tests, they are allowed to not do the work between the chapter tests.

Pre- and Post-Test of TAKS released items

The items from the pre- and post-test are categorized into the three main concepts: variables, expressions and equality. Appendix C contains all of the pre-test items, their assigned numbers on the pre-test, and their placement in the course for the post-test. Only 19 questions cover the concepts of variables, expressions, and equality that were used in this study. The questions that involve variables are 2, 12, 20, 22, 24, and 32. The

[32] This is from direct experience I had while working as the mathematics teacher at Gaggle HS for a summer. Many of the students had the material before and wanted to speed through the course.
[33] I will be interpreting the teacher's assessments of the students' work throughout the rest of this chapter.

questions that include expressions are 10, 23, 25, 26, and 27. The questions that comprise equality are 4, 8, 9, 29, 31, 34, 35, and 36.

Quantitative Results

The null hypothesis was that there would not be any difference in the scores of the pre- and post-test for the treatment group. The scores of the eight students who completed both tests are significantly different. The results of a Paired Sample T-test are in Table 3 (output from SPSS 11).

Table 3: Paired Sample Test of the Treatment Group

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	Pre-test score - Post-test score	-.5308	.14356	.05076	-.6508	-.4108	-10.458	7	.000

The t-value of -10.458 means that it is very unlikely that the differences in the treatment group's pre-test and post-test occurred by chance. The intervention most likely helped the students achieve the better scores on the post-test.

The results from the control group compared to the treatment group follow. The 19 questions were categorized by which TAKS objective and grade level the questions fell within; the results are presented by category in Table 5.

Using a null hypothesis that the Control Group's score will be greater than the Treatment Group's score and entering the data into a TI-84 calculator to use a Two Sample T-Test the following output was selected: Control group's scores < Treatment group's scores.

Table 4: Overall scores: Control group's scores < Treatment group's scores

	Two Sample T-Test				
	Means	Std. Deviation	t = – 11.9875	df = 20.511	Sig. .000 p=5.02E-11
Control group	$\overline{x_1}$ = .473	S_{x_1} = .1724		67	
Treatment group	$\overline{x_2}$ = .877	S_{x_2} = .0789		9	

This result seems to suggest that the treatment group's scores on the post-test were significantly better than the control group's scores. The small p-value suggests that this did not occur by chance.

Table 5: Output from TI-84 with data from the post-test

TAKS Objective and Grade Level	Question numbers from the Pre-Test	Test Case: Control < Treatment
7th Obj. 2: Patterns, Relationships and Algebraic Reasoning	Q29, Q31	p=0.00000005013
8th Obj. 1: Numbers, Operations, and Quantitative Reasoning	Q32	p=0.0000000000169234
8th Obj. 2: Patterns, Relationships and Algebraic Reasoning	Q4, Q12	p=0.16003
9th Obj. 1: Functional Relationships	Q2, Q8, Q9, Q24, Q35	p=0.00422
9th Obj. 2: Properties and Attributes of Functions	Q25, Q26, Q27	p=0.000089391
9th Obj. 3: Linear Functions	Q22	p=0.009256
9th Obj. 4: Linear Equations & Inequalities	Q10, Q20, Q23, Q36	p=0.00633
9th Obj. 10: Mathematical Processes and Tools	Q34	p=0.00762

The small p-values (except for 8th grade Obj.2) indicate that the events did not occur by chance. For seven of the eight objectives, the treatment group's scores were the same as or greater than the control group's scores. Was something other than chance involved? The answer to this will be discussed in the following chapter.

The quantitative data only tell part of the story. The observations made by the researcher throughout the 2004–2005 school help to explain how the significant differences could occur. The gap of how the post-test was administered to the treatment group influences the treatment group's scores tremendously. The ability of the treatment group to redo not only the open-ended questions in the GA but also the multiple-choice questions (which 19 were used for the post-test) was a major advantage.

In order to determine if the quantitative results show a true picture or not, then a finer look at the qualitative data will occur and follows in the next section.

Eight Case Studies

Ana M.

Ana M. started the ASK ME-Algebra course on December 1, 2004. When Ana took the pre-test of TAKS-like items, her score was 5 correct out of 19 questions. On the post-test of the same items, her score was 16 correct out of 19 questions. Ana had the next to the lowest score on the pre-test and she made the median score on the post-test. Ana M. did have questions 9 and 20 correct on the pre-test; 9 and 20 were categorized as dealing with both equality and with variables. Ana responded to questions 35 and 36 correctly on the pre-test; 35 and 36 assess the concept of equality. Out of the 11 questions that assessed variables, Ana answered 2 correctly on the pre-test. Ana answered none of the 5 questions about expressions correctly on the pre-test and out of the 10 questions that assessed equality on the pre-test, Ana answered 2 correctly.

Ana responded incorrectly to questions 10, 20, and 22 on the post-test; question 10 deals with expressions; questions 20 and 22 were categorized as dealing with both variables and equality. There were 11 questions that assessed variables; Ana answered 9 correct on the post-test. There were 5 questions that assessed expressions, Ana answered 4 correct on the post-test. There were 10 questions that assessed equality; Ana answered 8 correct on the post-test.

These data seem to indicate that Ana understands the concepts of variables, expressions, and equality better after working through the ASK ME-Algebra course then before taking the course.

Activity Information

Date Saved	Location	Title
Dec 02, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Dec 02, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Dec 02, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
Dec 15, 2004	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities
	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data
	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model
Feb 23, 2005	Unit 2: The Trash Problem	Activity 2: Defining Linear Functions
	Unit 2: The Trash Problem	Activity 3: Introduction to Motion

Figure 5: Ana M.'s activities that were saved.

Of the 11 Unit 1 activities, Ana saved her work in 4 of them. Of the 13 Unit 2 activities, Ana saved her work in 1 of the activities but not in any of the 15 activities in Unit 3. Ana did mention that she had viewed the function machine tutorial but that was the only tutorial she could identify from Unit 1 in her first interview.

Ana's grades on her Graded Assignments are as follows:

Table 6: Ana's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 94%, U1GA3, 87%, U1GA4, 100%	U2GA1, 87%, U2GA2, 100%, and U2GA3, 86%	
Expressions		U2GA4, 84% and U2GA5, 86%	U3GA2, 58%
Equality			U3GA1, 95% U3GA2, 58% and U3GA4, 91%
Other	U1GA1, 90% and U1GA5, 79%,	U2GA6, 60%	U3GA3, 89% and U3GA5, 71%

These data seem to suggest that Ana understands the meaning of variables and equality as assessed in the Graded Assignments. The same cannot be concluded for her understanding of expressions since none of her grades for expressions were above the 90's.

During the first interview, Ana confessed that she does not "like math since it has always been hard", and she "never made a grade above a 70" in any math class. Ana felt that "some people just know [algebra] and others that don't."

According to Ana, neither of her parents like math. Her father went to school until the 9th or 10th grade and her mother received a GED. Ana plans to attend a local community college to become a massage therapist. Her plans for future mathematics courses were to take Algebra II.

During the second interview, after asking Ana how she was doing in her algebra class, her response was, "I think I'm doing better than at the beginning because at the

beginning I really wasn't, you know, working as hard but I am working better now." By May 26, 2005, Ana had received credit for the course with 1356 points out of 1600 or 85%.

Ana admitted that, "I don't like math at all." Ana stated the concepts she understands the most are, "like the mean, median and mode stuff. Yeah, that's really easy to me", and the concept that is challenging for her is, "like, slope, I'm not good with slope."

When Ana was asked to solve the following question: 8 + 4 = □+ 5, she wrote 12 in the box. When asked, "*What did you think about when you picked 12?"* She replied, "Well, I just added the 8 plus 4."

Ana was asked to determine if the following was sometimes true, always true or never true: x – y = z – y. She replied, "Never." When asked, "*Why do you say that?"* She said she was "guessin'". She was asked to simplify, 3a – (b + a), her response was, "I don't know how to simplify. It's hard. I really don't. That's what I should have told you, I don't know how to do that."

After watching Ana attempt the tasks, my conclusion is that it does not seem that she has a complete understanding of variables, expressions or equality. The tasks given to Ana were out of context and on paper, so she might not have been prepared to work with those types of problems and in that medium. Ana took the TAKS test during April of 2005 and October of 2005. She did not pass the mathematics portion on either test. Ana withdrew from Gaggle HS on March 24, 2006 and transferred to a larger city's charter school. However, it appears that Ana has understood the concepts of variables

and equality better after treatment then she did before. I do not feel comfortable making that statement about her understanding of expressions.

Ana's views about mathematics changed during treatment, as well. Here are the graphs of each construct from VAMS:

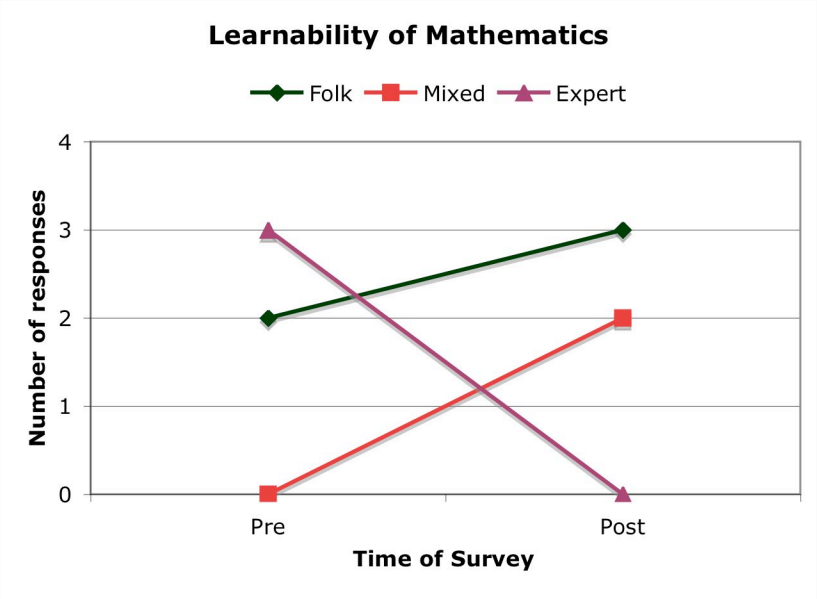


Figure 6: Ana's views of the "Learnability" of Mathematics

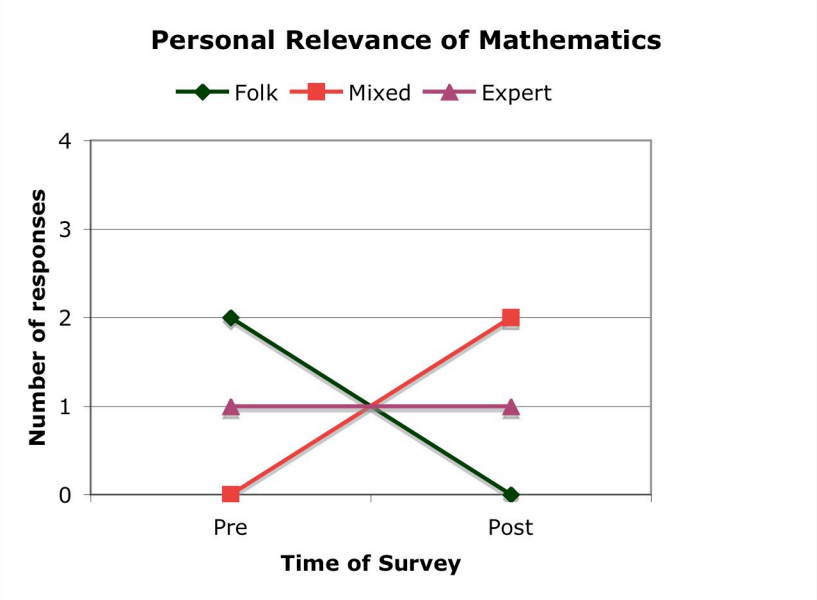


Figure 7: Ana's views about the Personal Relevance of Mathematics

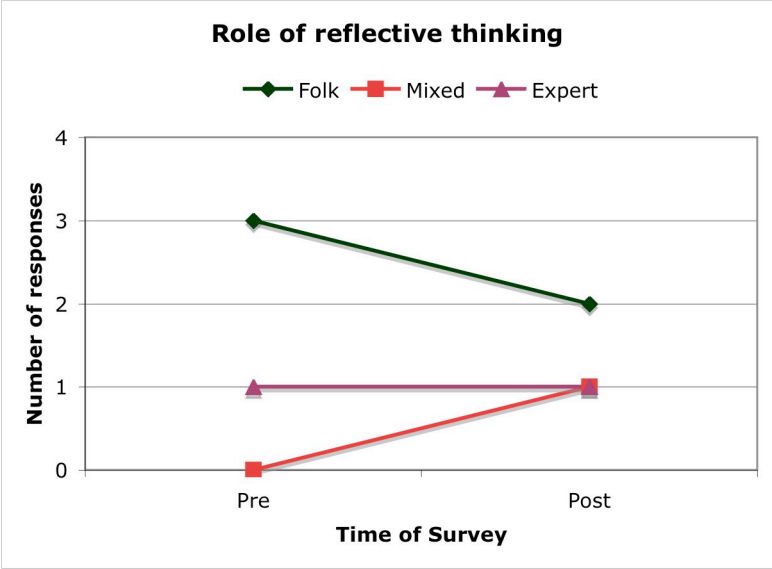


Figure 8: Ana's views of the Role of Reflective Thinking

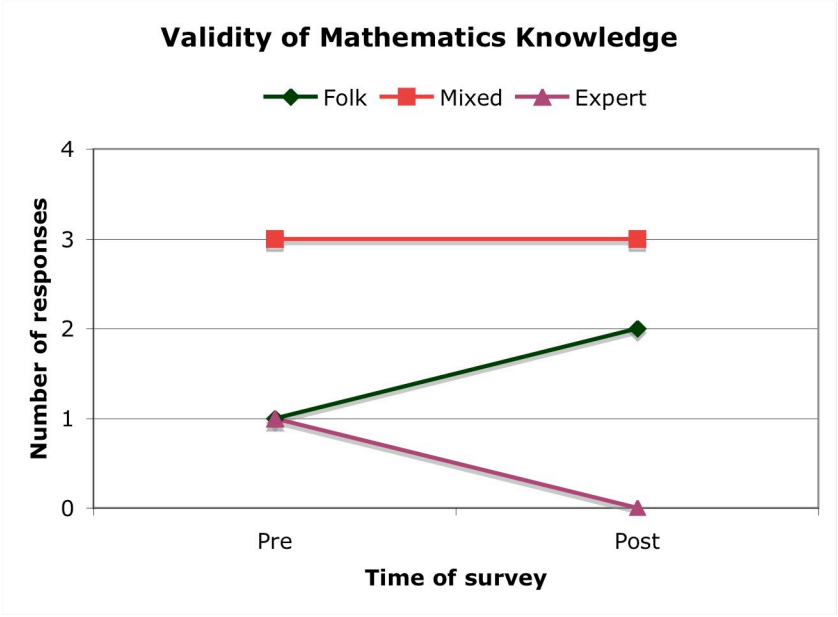


Figure 9: Ana's views of the Validity of Mathematics Knowledge

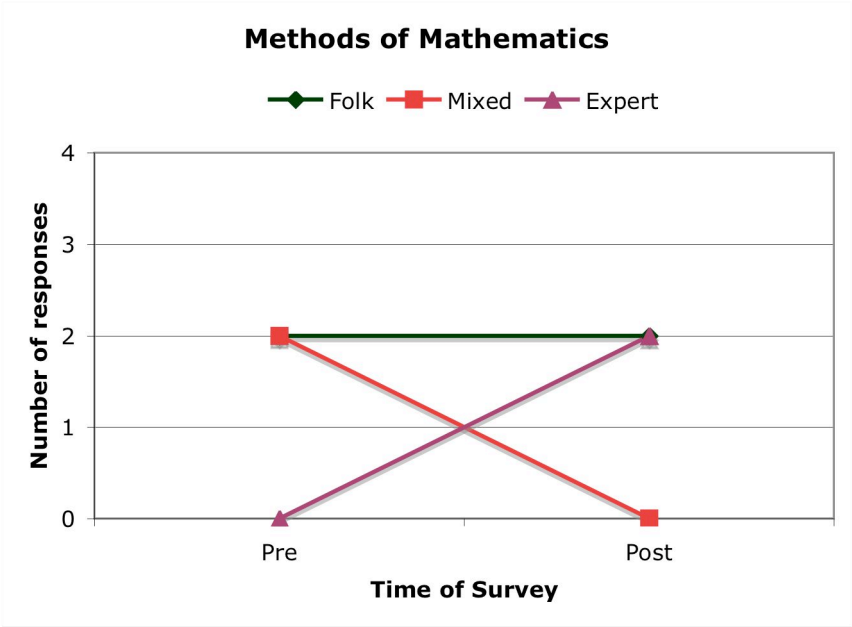


Figure 10: Ana's views of the Methods of Mathematics

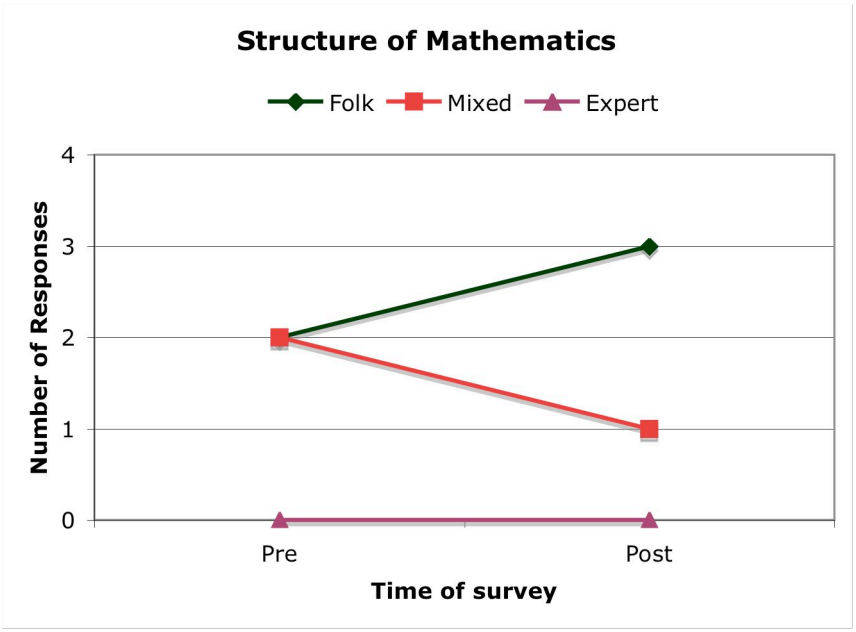


Figure 11: Ana's views of the Structure of Mathematics

Ana's views about mathematics did not improve during treatment. Overall, her Expert views either decreased or stayed the same and her Folk views stayed constant.

Bubba B.

Bubba B. started the course on September 15, 2004 and did not turn in his first assignment until October 22, 2004. When Bubba took the pre-test of TAKS-like items, his score was 6 correct out of 19 questions. The items Bubba answered correctly on the pre-test were 4, 12, 24, 25, 27, and 29. The questions that were categorized as involving variables that Bubba correctly answered were 4, 12, 25, and 27. Of the 11 questions that assessed variables, Bubba answered 4 correctly on the pre-test. Question 24 dealt with expressions. Bubba answered 1 out 5 questions about expressions correctly. Question 29 dealt with equality. Bubba answered 1 out of 10 questions correctly about equality.

On the post-test of the same items, his score was 18 correct out of 19 questions. Bubba correctly answered all 11 questions about variables on the post-test. Bubba correctly answered all 10 questions about equality. Bubba did have question 24 correct on the pre-test but not on the post-test. There are two possibilities that can explain why Bubba correctly answered question 24 before the course and incorrectly answered question 24 after the course. The first is that Bubba was guessing at the answers during the pre-test. The second is that Bubba understood expressions before taking the course and the material in the course confused him. The second possibility does not seem likely since of the 5 questions categorized to include expressions, Bubba only answered 1 question dealing with expressions correct on the pre-test.

These data seem to suggest that Bubba became better at variables, expressions and equality by working through the ASK ME-Algebra course.

Activity Information		
Date Saved	**Location**	**Title**
Sep 17, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Sep 17, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Sep 23, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
Sep 27, 2004	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Oct 04, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
Oct 04, 2004	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
Oct 06, 2004	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
Oct 07, 2004	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
Oct 11, 2004	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
Oct 28, 2004	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities

Figure 12: First part of Bubba B.'s saved work.

	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data
Jan 10, 2005	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model
	Unit 2: The Trash Problem	Activity 2: Defining Linear Functions
Jan 20, 2005	Unit 2: The Trash Problem	Activity 3: Introduction to Motion
Feb 25, 2005	Unit 2: The Trash Problem	Activity 4: Rate of Change I
Feb 25, 2005	Unit 2: The Trash Problem	Activity 5: Rate of Change II
Feb 24, 2005	Unit 2: The Trash Problem	Activity 6: The Linear Parent Function
	Unit 2: The Trash Problem	Activity 7: A Variation of the Linear Parent Function: y = mx

Figure 13: Second part of Bubba B.'s saved work.

Bubba saved his work in 10 of the 11 activities in Unit 1 but he saved his work in 5 of the 13 activities in Unit 2 and none in Unit 3. Bubba's grades on the Graded Assignments are:

Table 7: Bubba's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 88%, U1GA3, 96% and U1GA4, 93%	U2GA1, 93%, U2GA2, 97% and U2GA3, 86%	
Expressions		U2GA4, 91% and U2GA5, 100%	U3GA2, 77%
Equality			U3GA1, 85%, U3GA2, 77% and U3GA4, 92%
Other	U1GA1, 96% and U1GA5, 89%	U2GA6, 89%	U3GA3, 91% and U3GA5, 77%

These data seem to suggest that Bubba understands the concepts of variables, expressions and equality as assessed on the GA, especially since the lowest grade he made on a GA was a 77.

During the first interview Bubba acknowledged that he "does not like Algebra because it is hard" and he "thinks if someone is smart then they will learn algebra really fast and get it and once they learned it, they know it".

Bubba's mother went to college and his father graduated from high school. He plans to go to college in a nearby city. He also plans to take Geometry and Algebra II or "enough [math] to graduate".

During the second interview, after being asked how he was doing in the course, Bubba stated, "Well, I don't know. My step-dad has been helping me a lot. So, I think I'm doing good." He also admitted that the concepts he understands the best are: "just the x- and y-intercepts and the graph stuff."

When Bubba was asked to solve the following question: 8 + 4 = $\square$ + 5, he wrote 12 in the box. When asked what he did, he replied, "I added 8 and 4." Bubba was asked to determine if the following was sometimes true, always true or never true: x – y = z – y. He replied, "It's never true, right?" He was asked to simplify, 3a – (b + a). He replied, "That's a b? *Yes*. So, that's 3a minus b plus a? *Correct*. So, that's, oops, b plus 3a?"

This data seem to suggest that Bubba does not understand the concepts of variables, expressions and equality as assessed in the tasks. The tasks given to Bubba were out of context and on paper, so he might not have been prepared to work with those types of problems and in that medium. By May 26, Bubba completed the first semester of ASK ME-Algebra and accumulated 1440 out of 1600 points or 90%. Bubba transferred back to the non-accredited school, so he was not required to take another TAKS test.

Overall, I can conclude that Bubba has a better understanding of variables, expressions, and equality than he did before he started the ASK ME-Algebra course.

Bubba's views about mathematics changed during treatment, as well. Here are the graphs of each construct from VAMS:

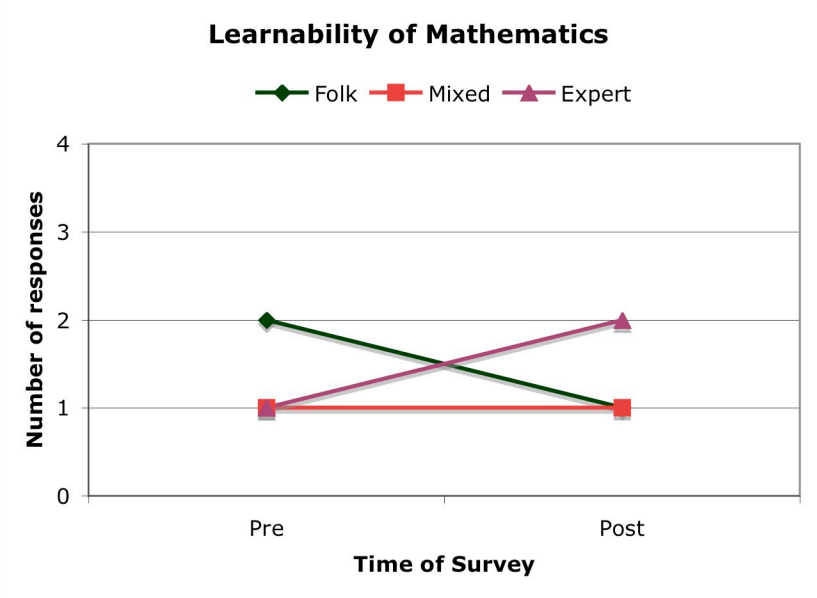


Figure 14: Bubba's views of the "Learnability" of Mathematics

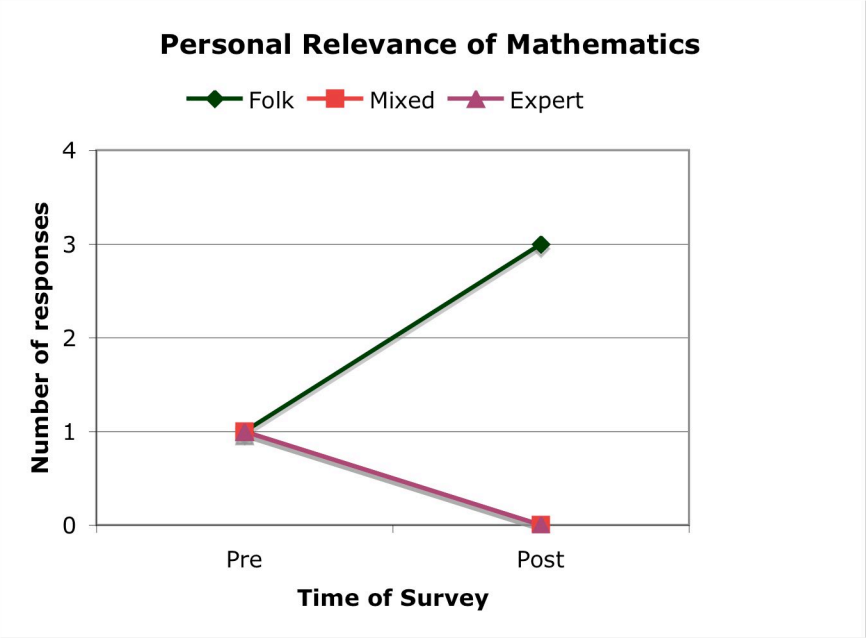


Figure 15: Bubba's views of the Personal Relevance of Mathematics

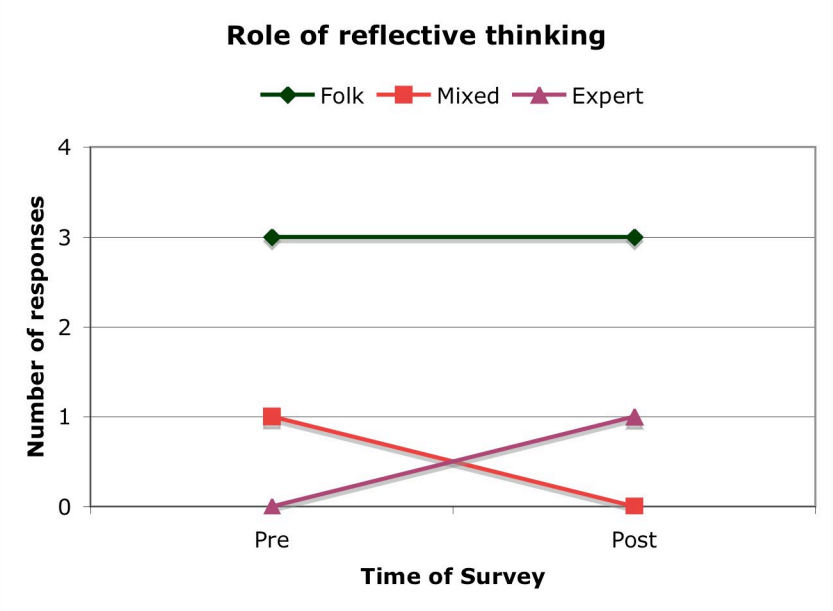


Figure 16: Bubba's views of the Role of Reflective Thinking

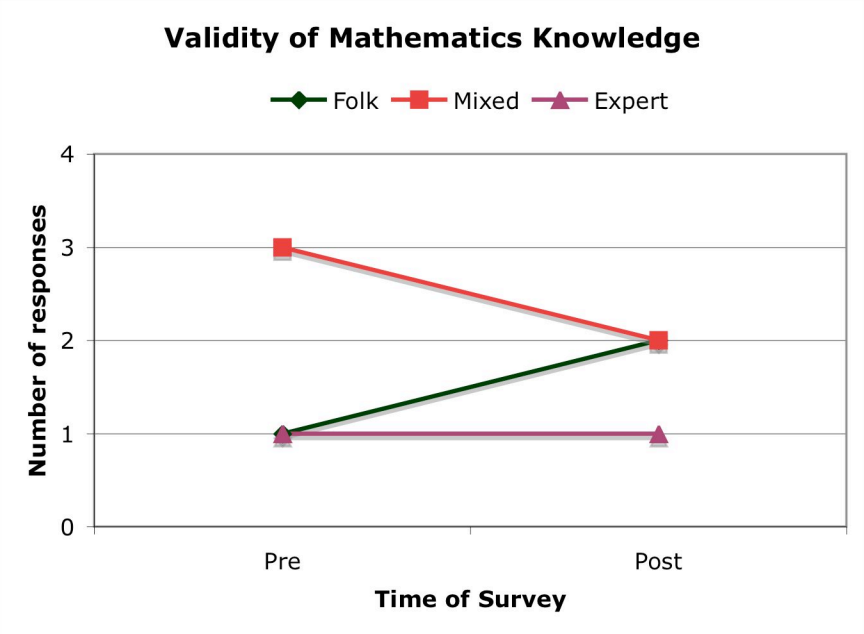


Figure 17: Bubba's view of the Validity of Mathematical Knowledge

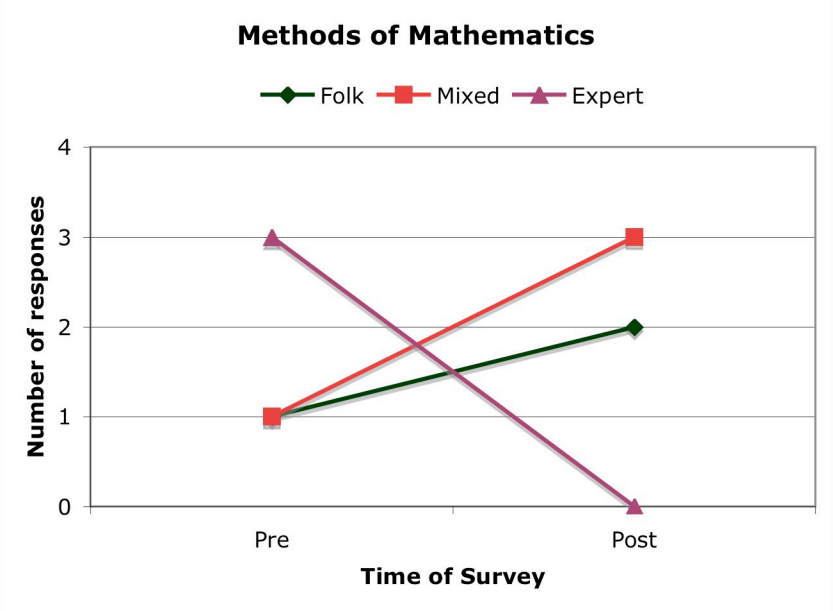


Figure 17: Bubba's views of the Methods of Mathematics

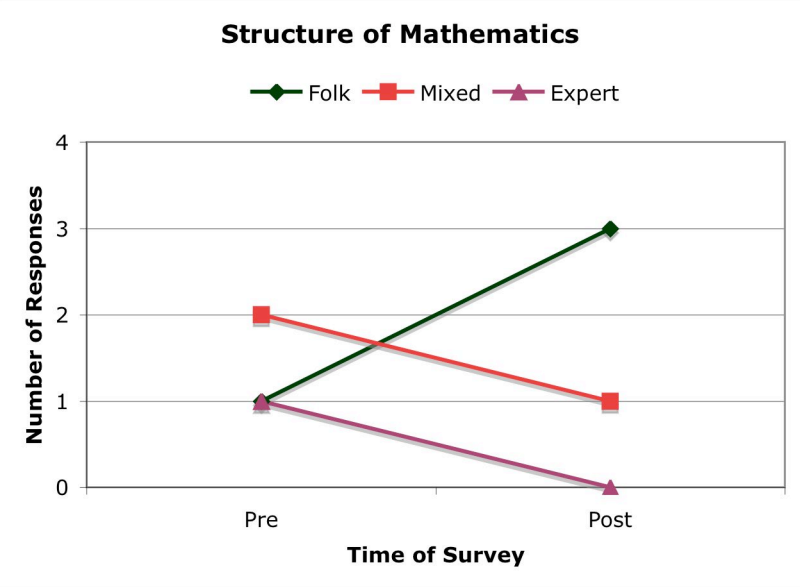


Figure 18: Bubba's views of the Structure of Mathematics

Bubba's views about mathematics changed during treatment. Four of the six constructs show Bubba's view changing negatively (i.e. Folk increasing and Expert decreasing): Personal Relevance, Validity, Methods and Structure.

Barbara W.

Barbara W. started the course on August 30, 2004. When Barbara took the pre-test of TAKS like items, her score was 8 correct out of 19 questions. Barbara answered 8, 9, 10, 22, 23, 26, 31, and 32 correctly on the pre-test. The questions categorized as dealing with variables were 8, 9, 22, and 26; Barbara answered 4 of 11 questions that assessed variables correctly on the pre-test. Of the 5 questions that assessed expressions, Barbara answered 3 correctly: 10, 23, and 32. Questions 8, 9, 22, and 31 are categorized as assessing equality; Barbara answered 4 out of 10 correctly on the pre-test.

On the post-test of the same items, her score was 17 correct out of 19 questions. Barbara answered 25 and 27 incorrectly on the post-test. Questions 25 and 27 were categorized as dealing with variables. On the post-test, Barbara answered 9 of the 11 questions that dealt with variable correctly. Barbara answered all 5 of the questions about expressions and all 10 of the equality questions correctly on the post-test.

These data seem to suggest that Barbara understands the concepts of variables, expressions and equality better on the post-test than on the pre-test.

Activity Information		
Date Saved	**Location**	**Title**
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Aug 30, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Aug 26, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
Sep 08, 2004	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Sep 07, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
Sep 09, 2004	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
Sep 13, 2004	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
Sep 13, 2004	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
Sep 14, 2004	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
Sep 15, 2004	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities
Sep 16, 2004	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data
Oct 14, 2004	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model
Nov 11, 2004	Unit 2: The Trash Problem	Activity 2: Defining Linear Functions
Nov 12, 2004	Unit 2: The Trash Problem	Activity 3: Introduction to Motion

Figure 19: Barbara W.'s saved activities.

Barbara saved her work in all of the Unit 1 activities and she saved her work in 3 of the 13 activities in Unit 2 but none of her work in Unit 3 was saved. Barbara's grades from her GA are as follows:

Table 8: Barbara's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 90%, U1GA3, 94% and U1GA4, 100%	U2GA1, 95%, U2GA2, 97% and U2GA3, 96%	
Expressions		U2GA4, 100% and U2GA5, 90%	U3GA2, 100%
Equality			U3GA1, 94%, U3GA2, 100% and U3GA4, 100%
Other	U1GA1, 100% and U1GA5, 93%	Unit 2GA6, 77%	U3GA3, 99% and U3GA5, 83%

These data seem to suggest that Barbara does understand the concepts of variables, expressions, and equality as assessed in the Graded Assignments.

Barbara said, "if I work hard then I get good grades". She would feel guilty whenever she made bad grades. At the time of the first interview, Barbara planed to take Math Models with Applications[34] instead of Algebra II.

During the first interview, Barbara disclosed that, "algebra is complicated and aggravating". She heard that her father loved math but she knows that her mother does not like math. Her mother did not finish high school. Barbara plans to go to college to become a massage therapist.

By the second interview, Barbara had completed the first semester of Algebra I and received credit. When asked what grade she received, she replied, "93." She

[34] This course is an option for students who are not going to college but still need a third year of mathematics credit to graduate from high school. It is typically taken after Geometry during a student's junior or senior year of high school.

actually received 94% for the first semester grade on March 9, 2005, which was an accumulation of 1580 points out of 1600.

Barbara stated she understood "[t]he linear functions and all that stuff. I was good at that. 'Cause it was the easiest part for me. And that was what the whole thing was about, pretty much." When Barbara was asked to solve the following question: 8 + 4 = $\square$ + 5, she replied, "12." She explained what she did, "I just added 8 and 4 together." Barbara finished the GA that covered equations on March 9 and her second interview was conducted on March 22. Barbara was asked to determine if the following was sometimes true, always true or never true: x – y = z – y. She replied, "Never be true." When asked, *"Why do you say that?"* She replied, "I'm not sure." She was asked to simplify, 3a – (b + a). She replied, "Nope. I'm a little out of it. I know how to do it, I know you're supposed to plug everything in, aren't you supposed to use like a, the a, b, c, square thing or whatever?" The interviewer stated, "*Oh, no. You don't need to do that. I just want you to combine any like terms."* Barbara continued, "Oh, the a's. So, like 3a, (pauses) plus b."

These data seem to suggest that Barbara does not understand the concepts of variables, expressions, and equality as assessed in the tasks. The tasks given to Barbara were out of context and on paper, so she might not have been prepared to work with those types of problems and in that medium. Barbara took the TAKS test during April 2005, July 2005, and October 2005 but she did not pass the mathematics portion on any of the tests. She is signed up to take the test during April 2006. On another note, her baby was due any day at the beginning of March 2006.

Overall, I can conclude that Barbara has a better understanding of variables, expressions and equality than she did before taking the ASK ME-Algebra course since she did well on the post-test and on the GA that assessed the concepts.

Barbara's views about mathematics changed during treatment, as well. Here are the graphs of each construct from VAMS:

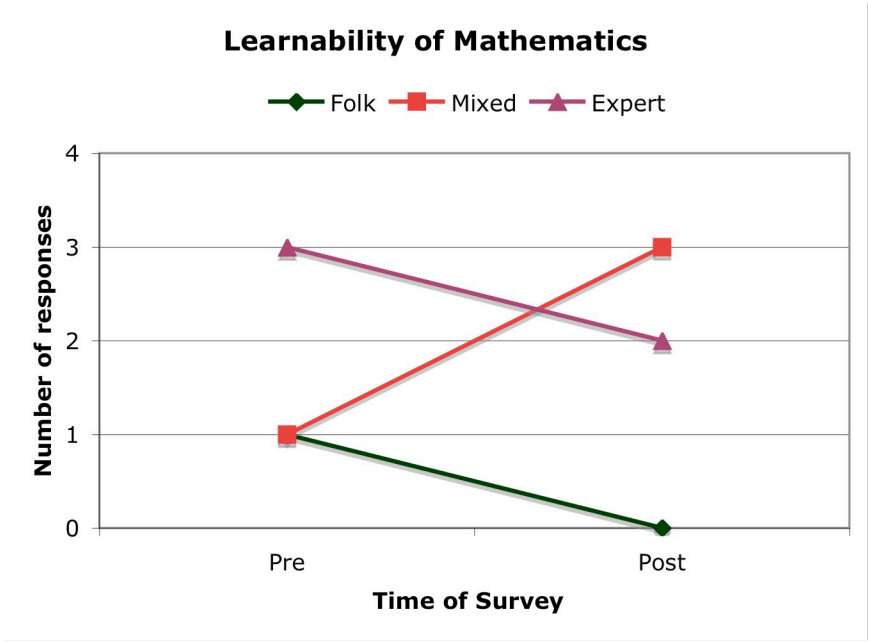


Figure 20: Barbara's views of the "Learnability" of Mathematics

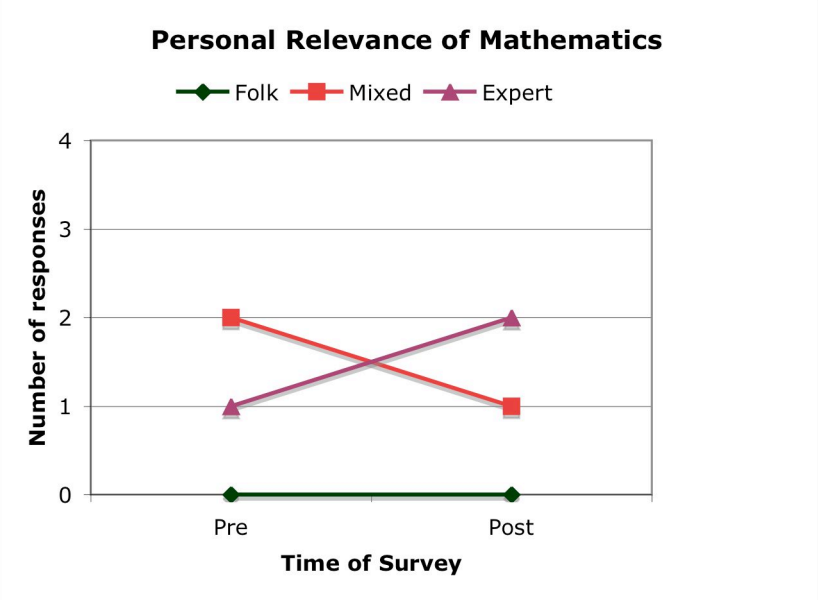


Figure 21: Barbara's views of the Personal Relevance of Mathematics

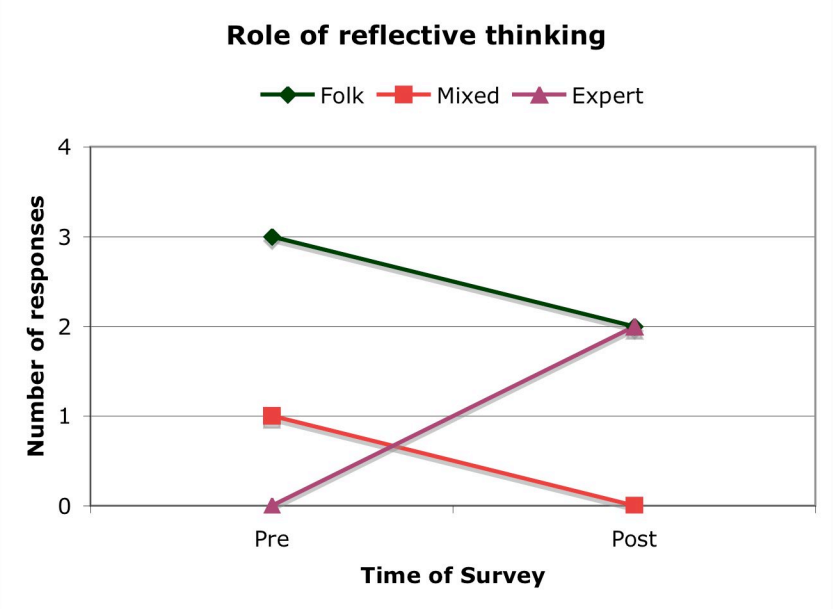


Figure 22: Barbara's views of the Role of Reflective Thinking

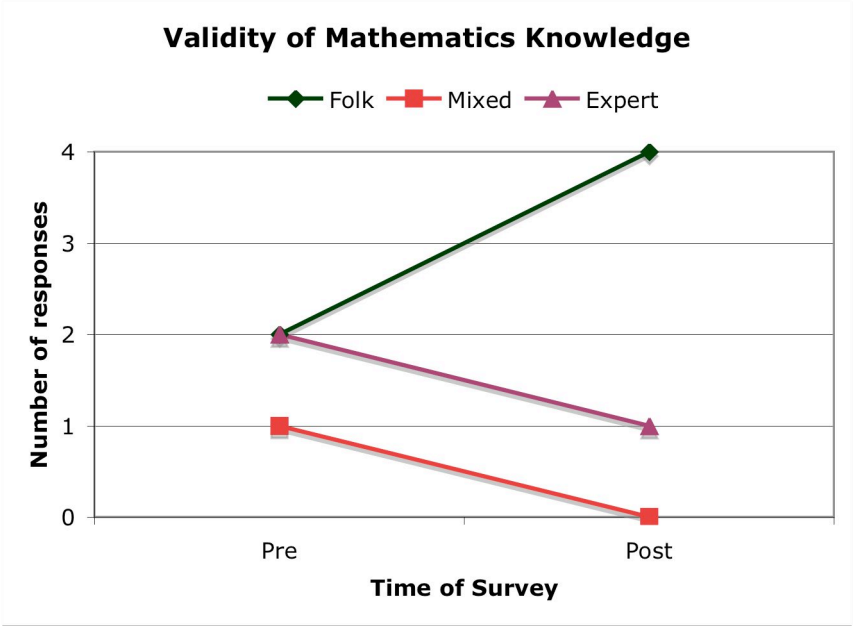


Figure 23: Barbara's views of the Validity of Mathematics Knowledge

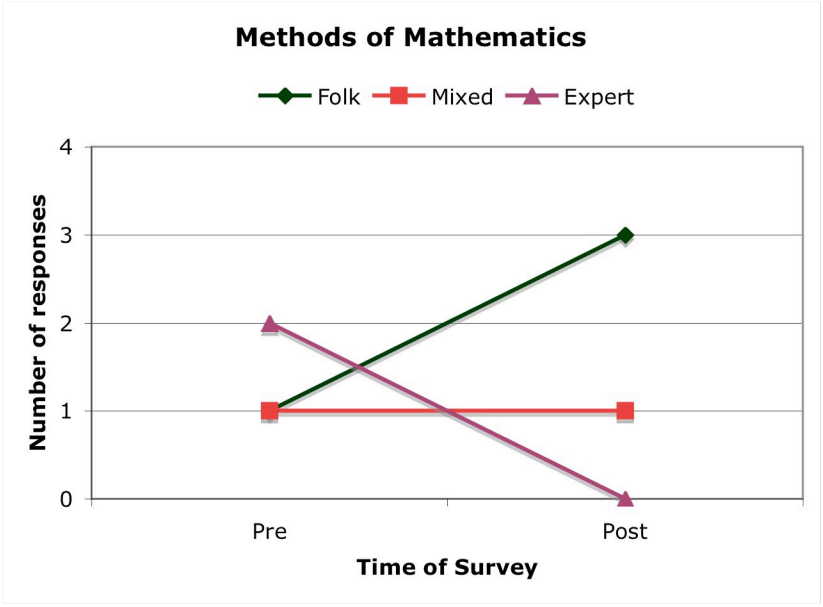


Figure 24: Barbara's views of the Methods of Mathematics

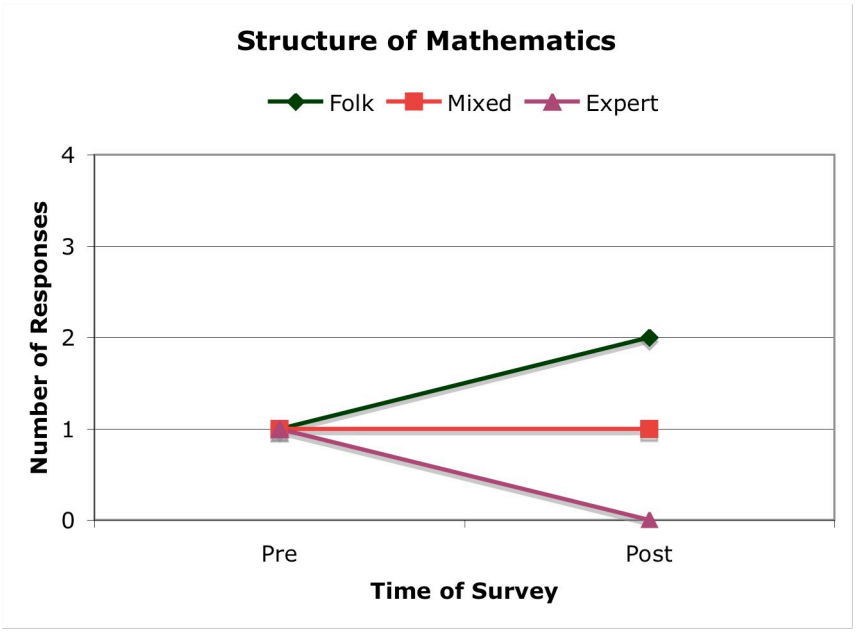


Figure 25: Barbara's views of the Structure of Mathematics

Barbara's views about mathematics did not improve after treatment. Her views worsen in three of the six constructs: Validity, Methods, and Structure.

Günter G.

Günter G. started the course on September 17, 2004. When Günter took the pre-test of TAKS-like items, his score was 7 correct out of 19 questions. Günter answered the following questions correctly on the pre-test: 4, 10, 23, 25, 32, 35 and 36. The questions that covered variables were 4 and 25. Günter correctly answered two out of 11 questions about variables. The questions that covered expressions were 10, 23 and 32. Günter answered 3 out of 5 correctly about expressions. The questions that covered equality were 4, 35 and 36. Günter answered 3 out 10 questions correctly about equality.

On the post-test of the same items, his score was 16 correct out of 19 questions. Günter G. answered 24, 25 and 27 incorrectly on the post-test. Questions 25 and 27 were

categorized as dealing with variables. Günter answered 8 out of 11 correctly about variables on the post-test. Question 24 was categorized as dealing with expressions. Günter correctly answered 4 out of 5 items about expressions on the post-test. Günter did not miss any of the 10 questions on equality. These data seem to suggest that Günter understands variables, expressions and equality better than he did before working through the ASK ME-Algebra course.

Activity Information		
Date Saved	**Location**	**Title**
Sep 27, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Aug 25, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Aug 25, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
Sep 09, 2004	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Sep 07, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
Sep 09, 2004	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
Sep 13, 2004	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
Sep 16, 2004	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
Sep 20, 2004	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
Sep 23, 2004	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities
	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data

Figure 26: Günter G.'s saved work.

Günter only saved his work in the first 10 activities in Unit 1. He did not save any work in Unit 2 or in Unit 3. Günter's grades from his GA are as follows:

Table 9: Günter's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 98%, U1GA3, 98% and U1GA4, 100%	U2GA1, 99%, U2GA2, 93% and U2GA3, 94%	
Expressions		U2GA4, 100% and U2GA5, 99%	U3GA2, 91%
Equality			U3GA1, 93%, U3GA2, 91% and U3GA4, 84%
Other	U1GA1, 99% and U1GA5, 95%	U2GA6, 67%	U3GA3, 82% and U3GA5, 72%

These data seem to suggest that Günter does understand variables, expressions and equality as assessed in the GA.

Günter stated during the first interview that algebra is "a lot complicated" and "it's kind of hard to understand". He also thinks that someone who is good at math needs "a lot of practice".

Günter's mother likes math and both of his parents graduated from high school. He plans to go to college or technical school to study architecture. His plan for a future mathematics class is to take Mathematical Models with Applications.

During the second interview, Günter was asked to find the value that goes in the box: $8 + 4 = \square + 5$. He replied, "(wrote 7 on his paper) Would it be that?" *Tell me what you did*. "I subtracted 4 from this side and then I had to subtract 4 from that side to get rid of this 4 right there. And then I had 8 and then the box right there plus one, so I figured that it would be 7."

He was then asked if x – y = z – y true always, never, or sometimes? He responded, "Um, never. Just a guess." He was asked to simplify 3a – (b + a). His response was, "(Pauses) I don't remember doing this but (pauses). Oh." *Okay, tell me what you did.* Günter replied, "Um, you got to do this – yeah distribute." *Is there anything else you can do to simplify that?* "I guess you could put the …" *If you've distributed then can you simplify this step?* "Oh, yes, by putting the -3a with that." He wrote, 3a – 3b + -3a.

This data suggests that Günter does have a relational understanding of equality and a partial understanding of expressions. By April 6, 2005, Günter completed the first semester of Algebra and had accumulated 1464 points out of 1600 or 92%. When he was asked what his final grade was in the class, he responded, "It said an 85 or a 89 or something." Günter took the TAKS test during April 2005 but he did not pass the mathematic portion. He did not return to Gaggle HS for the 2005-2006 school year.

Günter does have a better understanding of the concepts of variables, expressions and equality than before he took the ASK ME-Algebra course since he scored better on the post-test than the pre-test, his GA scores were high, he correctly worked the interview task about equality and he almost correctly worked the interview task about expressions.

Jean C.

Jean C. started the course on August 27, 2004. When Jean took the pre-test of TAKS like items, her score was 3 correct out of 19 questions. Questions 2, 23 and 24 were answered correctly on the pre-test. Question 2 was categorized as dealing with variables and questions 23 and 24 dealt with expressions. Jean answered 1 out of 11

questions about variables correctly and Jean answered 2 out of 5 questions about expression correctly on the pre-test.

On the post-test of the same items, her score was 18 correct out of 19 questions. Jean C. did have question 35 correct on the pre-test but not on the post-test. Question 35 was categorized as dealing with equality. Jean answered 9 of the 10 questions about equality correctly, and she answered all 11 questions about variables correctly, and answered all 5 questions about expressions correctly. Jean's test scores were the most improved from 16% correct to 95% correct. These data seem to suggest that Jean has a better understanding of variables, expressions and equality than she did before taking the ASK ME-Algebra course.

Activity Information		
Date Saved	**Location**	**Title**
Aug 20, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
Sep 09, 2004	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Sep 02, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
Sep 23, 2004	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers

Figure 27: The first part of Jean C.'s saved work.

	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model
Nov 11, 2004	Unit 2: The Trash Problem	Activity 2: Defining Linear Functions
Nov 12, 2004	Unit 2: The Trash Problem	Activity 3: Introduction to Motion
Nov 17, 2004	Unit 2: The Trash Problem	Activity 4: Rate of Change I
	Unit 2: The Trash Problem	Activity 5: Rate of Change II

Figure 28: The second part of Jean C.'s work.

	Unit 3: The Animal Shelter Problem	Activity 3: A First Look at Inequalities
Jan 06, 2005	Unit 3: The Animal Shelter Problem	Activity 4: Solving Equations and Inequalities Using Tables and Graphs
	Unit 3: The Animal Shelter Problem	Activity 5: The Distributive Property
	Unit 3: The Animal Shelter Problem	Activity 6: Combining Like Terms
	Unit 3: The Animal Shelter Problem	Activity 7: Adding and Subtracting Expressions
Jan 31, 2005	Unit 3: The Animal Shelter Problem	Activity 8: Equations of the form x + c = k
	Unit 3: The Animal Shelter Problem	Activity 9: Equations of the Form kx = w

Figure 29: The final part of Jean C.'s work.

Jean saved her work in 6 of the 11 activities of Unit 1, in 3 of the 13 activities of Unit 2 and in 2 of the 15 activities in Unit 3. Jean's grades from her Graded Assignments that assessed variables are as follows:

Table 10: Jean's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 94%, U1GA3, 95% and U1GA4, 92%	U2GA1, 95%, U2GA2, 93% and U2GA3, 81%	
Expressions		U2GA4, 86% and U2GA5, 99%	U3GA2, 100%
Equality			U3GA1, 92%, U3GA2, 100% and U3GA4, 98%
Other	U1GA1, 97% and U1GA5, 92%	U2GA6, 93%	U3GA3, 95% and U3GA5, 81%

These data seem to suggest that Jean understands the concepts of variables, expressions and equality as assessed by the Graded Assignments; especially since her lowest grade was an 81.

Jean admitted during the first interview that she uses algebra in her science class. Jean liked Algebra when it was easy but once it became hard, she became lazy. Her dad uses mathematics as a carpenter but "he doesn't like it." Neither of her parents finished high school. Jean plans to attend college in Nebraska.

By the second interview (Jean was interviewed on March 22), Jean had completed the first semester of ASK ME-Algebra and had received credit. When asked what grade she received, she replied, "I don't remember my grade for it." She had received 93% for the first semester grade on March 9, 2005, which was an accumulation of 1483 points out of 1600 points.

Jean admitted that, "I still hate it", referring to algebra. She also admitted that her plans for the future had changed[35], "Cause, right now, I'm fixin' to have a kid, and you know, I don't really feel like taking a kid way up there (Nebraska)." She does plan to take Algebra II after Geometry.

When Jean was asked to solve the following question: 8 + 4 = □ + 5, she replied, "Let's see. Seven." Her explanation was, "I added this up here and see what this made and kind of, trying to see what would go here to make that." Jean was asked to determine if the following was sometimes true, always true or never true: x – y = z – y. She replied, "I'm guessing, never true?" When asked to simplify 3a – (b + a) Jean stated, "Oh, that looks hard. (pauses) I don't think that I can do this one. That's an "a" right?" *Yes*. "Let's see, if I put this (mumbles). Every thing I come out with, nothing comes from this side. I think you would have to divide here." *Well, that is not an equal sign that is a minus sign*. "Yeah. That's the problem. There's no equals."

These data seem to suggest that Jean does have a relational understanding of equality but not an understanding of variables and expressions. Jean took the TAKS test during April 2005 and she did pass the mathematics portion.

Jean does have a better understanding of variables, expressions, and equality than she did before she started the ASK ME-Algebra course since she scored better on the post-test than the pre-test, her GA scores were very high and she correctly worked the interview task.

[35] She had planned to go to college in Lincoln, Nebraska after graduating from Gaggle High School.

Jean's views about mathematics changed during treatment. Here are the graphs of her views of the six constructs of VAMS:

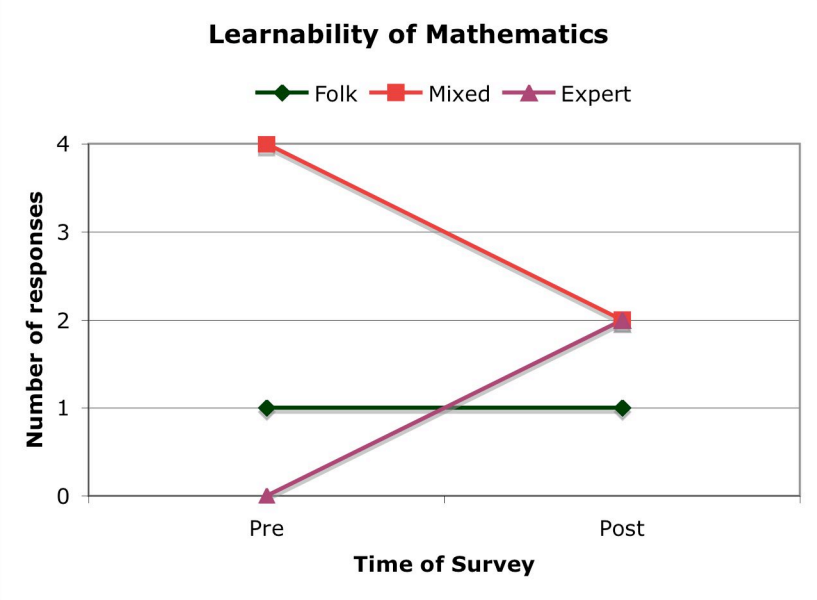


Figure 30: Jean's views about the "Learnability" of Mathematics

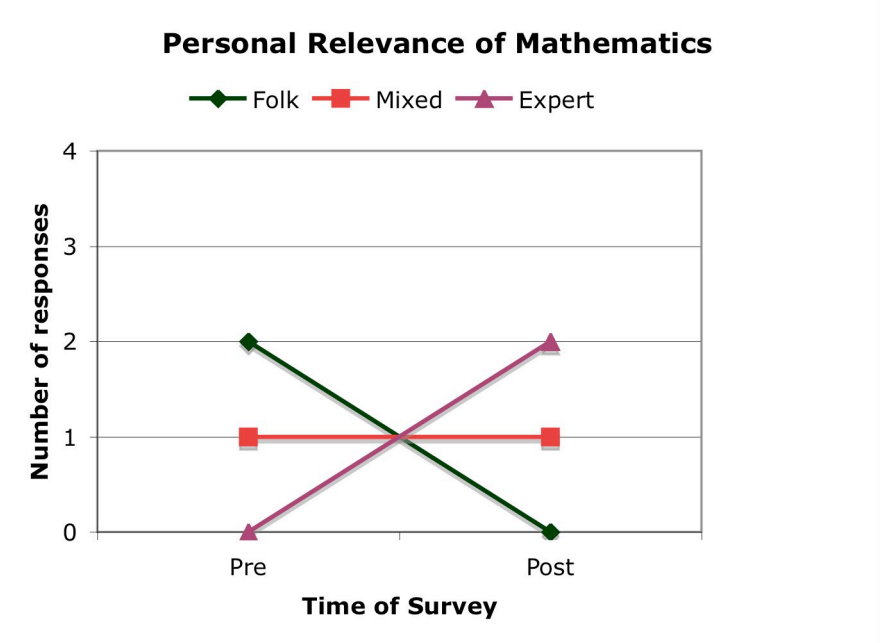


Figure 31: Jean's views of the Personal Relevance of Mathematics

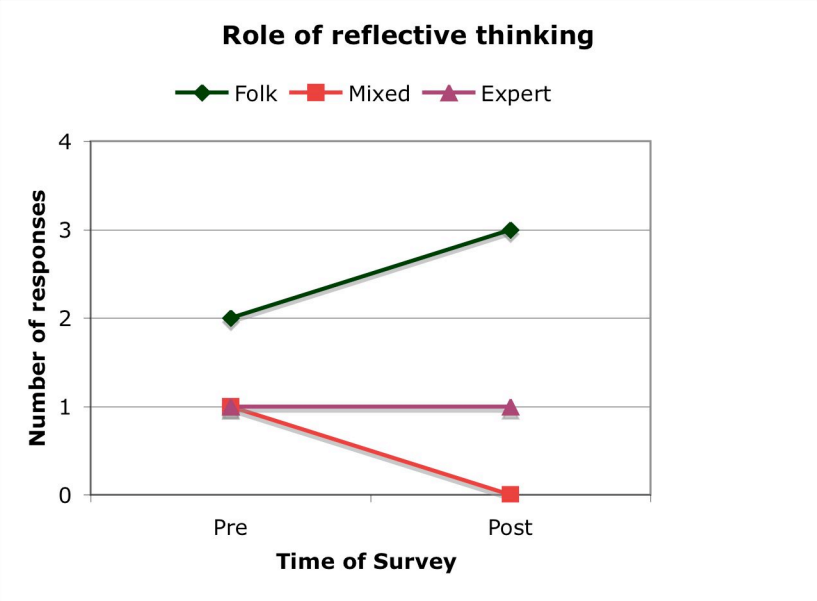


Figure 32: Jean's views of the Role of Reflective Thinking

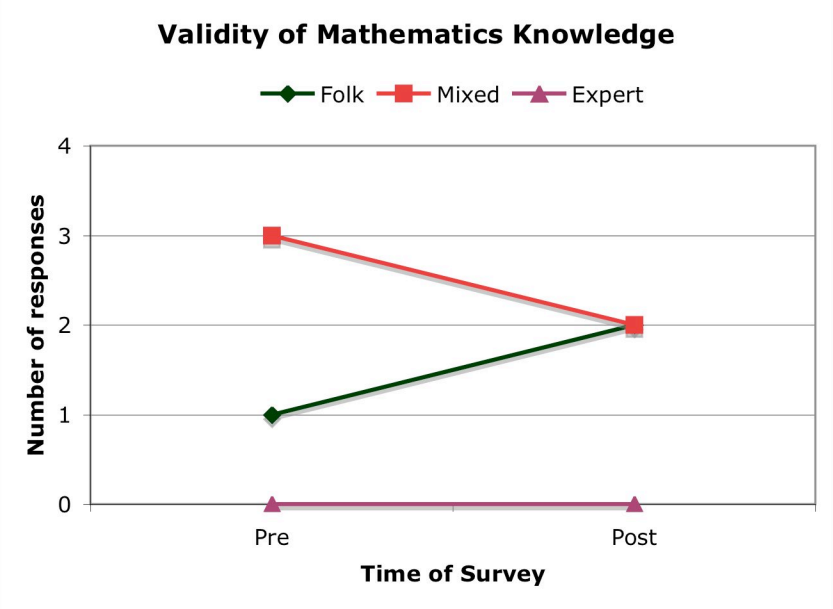


Figure 33: Jean's views of the Validity of Mathematics Knowledge

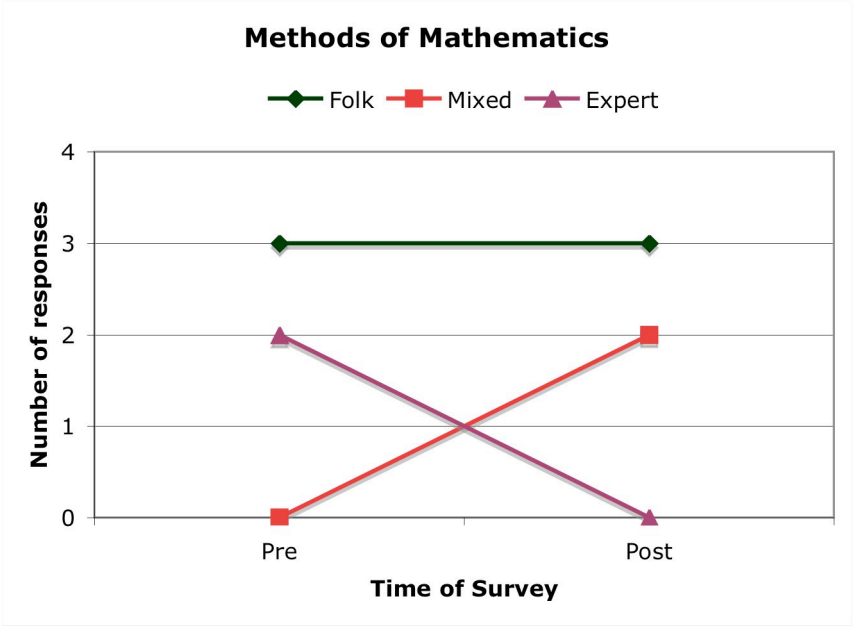


Figure 34: Jean's views of the Methods of Mathematics

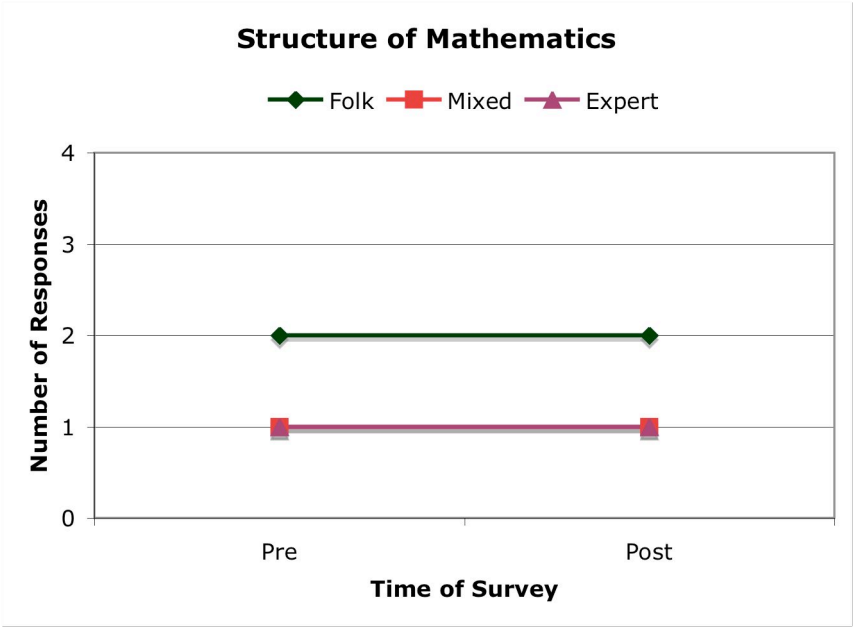


Figure 35: Jean's views of the Structure of Mathematics

Jean's views seem to be the most positive after treatment. Only her views of the Methods of Mathematics have decreased (i.e. Expert view decreased). She also had the most positive views before starting and during treatment.

Mary G.

Mary G. started the course on August 24, 2004. When Mary took the pre-test of TAKS-like items, her score was 3 correct out of 19 questions. Mary answered questions 10, 26, and 35 correctly on the pre-test. Question 10 was categorized as expressions; question 26 was from the variables category and question 35 is from the equality category. Mary answered 1 correctly out of 11 questions about variables, answered 1 correctly out of 5 questions about expressions, and answered 1 correctly out of 10 questions about equality.

On the post-test of the same items, her score was 14 correct out of 19 questions. The questions Mary answered incorrectly on the post-test were 4, 24, 32, 34, and 35. Mary answered 10 of 11 variable questions correctly on the post-test. She answered 2 of 5 expression questions correctly and 8 of 10 equality questions correctly. These data seem to suggest that Mary understands variables and equality well but not the concept of expressions after working through the ASK ME-Algebra course.

When asked how she was doing in the class so far she replied, "Not good. I guess for what I have done I got A, what I have done, I've done good but that was, somebody was helping me. And, they are not helping me anymore." She had not completed the course as of 11/2/2005 and she had accumulated 850 points from a total of 1600 or 53%. Mary did have a grade of 95% for what she had completed (nine of the sixteen graded assignments).

Activity Information		
Date Saved	**Location**	**Title**
Aug 20, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Aug 23, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Aug 26, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
Oct 18, 2004	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities
Sep 10, 2004	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data
	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model

Figure 36: Mary G.'s saved work.

Mary saved her work in 6 of the 11 activities in Unit 1 and she did not saved any work in Unit 2 or Unit 3. Mary's grades for the Graded Assignments that deal with variables are as follows:

Table 11: Mary's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 100%, U1GA3, 93% and U1GA4, 100%	U2GA1, 96%, U2GA2, 97% and U2GA3, 77%	
Expressions		U2GA4, 58% and U2GA5, 99%	U3GA2, 66%
Equality			U3GA1, 93%, U3GA2, 66% and U3GA4, Incomplete
Other	U1GA1, 93% U1GA5, 79%	U2GA6, Incomplete	UGA3, Incomplete and U3GA5, Incomplete

These data seem to suggest that Mary does understand variables as assessed in the Graded Assignments but that conclusion cannot be drawn for the concepts of expressions and equality.

During the first interview Mary admitted, "some people just know how to do it and some can't get it, I guess." Both of her parents "don't like math". Her father finished school when he was 23 and her mother never finished. Mary plans to go to tattoo school and get her license. She said that her plan for a future math class is to take "which ever one is easy".

During the second interview, Mary admitted, "I don't understand the word problems. I could do regular [math] problems. I can graph a line and make a table."

When Mary was asked to solve the following question: $8 + 4 = \square + 5$, she replied, "Do I put plus five in there too or just what that is?" She put 17 in the box.

Mary's second interview was conducted on May 10 and she had not attempted any of the graded assignments dealing with equations.

When asked if $x - y = z - y$ is always, never, or sometimes true, she wrote her response down, "Never". When asked to simplify $3a - (b + a)$, she replied, "Hmm, I don't know how."

These data seem to suggest that Mary does not understand variables, expressions or equality as assessed by the tasks. Mary took the TAKS test during April 2005 and October 2005 but did not pass the mathematics portion of either test. She has signed up to take the TAKS test during April 2006.

Mary understands the concept of variables better than she did before taking the ASK ME-Algebra course. Mary does not have an improved understanding of expressions or equality.

Nan M.

Nan M. started the course on August 30, 2004. When she took the pre-test her score was 5 correct out of 19 questions. Nan answered 4, 9, 10, 32, and 36 correctly. Nan answered 3 out of 11 questions correctly on the pre-test about variables, 2 out of 5 correctly about expressions and 3 out of 10 correctly about equality.

On the post-test of the same items, her score was 16 correct out of 19 questions. Nan M. answered 24, 25, and 26 incorrect on the post-test. Question 24 was categorized as dealing with expressions, and questions 25 and 26 were categorized as dealing with variables. Nan answered 9 out of 11 questions correctly on the post-test that assessed variables, 4 out of 5 questions correctly that assessed expressions and she answered all 10

questions correctly that assessed equality. These data seem to suggest that Nan improved her understanding about variables, expressions and equality as assessed on the pre- and post-test after working through the ASK ME-Algebra course.

Activity Information		
Date Saved	**Location**	**Title**
	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Aug 23, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Aug 23, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
Aug 24, 2004	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
Aug 25, 2004	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
Aug 30, 2004	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
Aug 30, 2004	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
Sep 07, 2004	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities
Sep 07, 2004	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data
Sep 13, 2004	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model
Sep 13, 2004	Unit 2: The Trash Problem	Activity 2: Defining Linear Functions
Sep 15, 2004	Unit 2: The Trash Problem	Activity 3: Introduction to Motion

Figure 37: The first part of Nan M.'s saved work.

	Unit 2: The Trash Problem	Activity 4: Rate of Change I
	Unit 2: The Trash Problem	Activity 5: Rate of Change II
Sep 21, 2004	Unit 2: The Trash Problem	Activity 6: The Linear Parent Function
Sep 29, 2004	Unit 2: The Trash Problem	Activity 7: A Variation of the Linear Parent Function: y = mx
	Unit 2: The Trash Problem	Activity 8: Another Variation of the Linear Parent Function: y = mx + b
	Unit 2: The Trash Problem	Activity 9: Writing Rules Given the Slope and y-intercept
	Unit 2: The Trash Problem	Activity 10: Writing Rules Given Points on the Line
	Unit 2: The Trash Problem	Activity 11: Using Function Notation
	Unit 2: The Trash Problem	Activity 12: Practicing Writing Rules
Oct 06, 2004	Unit 2: The Trash Problem	Activity 13: Practicing Linear and Non-Linear Functions
	Unit 3: The Animal Shelter Problem	Activity 1: Exploring Functions and Equations

Figure 38: The second part Nan M.'s saved work.

Nan saved her work in 10 of the 11 activities in Unit 1 and in 6 activities in Unit 2 but no activities were saved in Unit 3. Nan's grades for the GA are as follows:

Table 12: Nan's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 89%, U1GA3, 98% and U1GA4, 97%	U2GA1, 89%, U2GA2, 100% and U2GA3, 94%	
Expressions		U2GA4, 96% and U2GA5, 96%	U3GA2, 92%
Equality			U3GA1, 80%, U3GA2, 92% and U3GA4, 96%
Other	U1GA1, 96% and U1GA5, 87%	U2GA6, 86%	U3GA3, 98% and U3GA5, 81%

These data seem to suggest that Nan has an understanding of variables, expressions and equality as assessed by the Graded Assignments.

During the first interview Nan said, "when I attend class, I make As and Bs." She also admitted, "I am a poor student, I am not very good at math." Nan's mother does not like mathematics. Nan plans to attend a cosmetology school once she graduates from high school.

By the second interview, Nan had completed the first semester of ASK ME-Algebra and had received credit. When asked what grade she thought she received, Nan replied, "Hmm, no. Probably, probably like, probably in the 80's. I don't remember them." She had received 92% for the first semester grade on March 7, 2005, which was an accumulation of 1475 points out of 1600.

Nan was working through Algebra II for credit in a textbook. She admitted that she does not like Algebra I on the computer but she does like Algebra II in the book. "It [Algebra I] seems like it's harder than from out of the book. Or harder to understand."

When Nan was asked to solve the following question: 8 + 4 = □+ 5, she replied, "Plus five, what's that? Eight plus four equals twelve plus five. Am I suppose to get an answer after this?" She wrote 12 in the box. Nan, interviewed on March 22, had finished the GA dealing with equations on March 2. Nan was asked to determine if the following was sometimes true, always true or never true: x – y = z – y. She replied, "Never." When asked why she said never, she explained, "I just made a guess." When asked to simplify 3a – (b + a), she let out a big sigh and did not answer.

These data seem to suggest that Nan does not understand the concepts of variables, expressions and equality as assessed by the tasks. Nan took the TAKS test during April 2005 and passed the mathematics portion. She also graduated from Gaggle HS in May 2005.

Overall, Nan has a better understanding of variables, expressions and equality than before she took the ASK ME-Algebra course since she improved on the post-test and her GA scores were high.

Nan did not complete the final VAMS survey.

Vince P.

Vince P. started the course on August 24, 2004. When Vince took the pre-test of TAKS like items, his score was 6 correct out of 19 questions. Vince answered 4, 10, 20, 27, 29 and 35 correctly on the pre-test. He answered 2 out of 11 correctly about variables, 1 out of 5 correctly about expressions, and 4 out of 10 correctly about equality.

On the post-test of the same items, his score was 19 correct out of 19 questions. Vince answered all 11 correct that assessed variables, all five correct that assessed

expressions and 10 out 10 correct that assessed equality. These data seem to suggest that Vince understands variables, expressions and equality as assessed by the pre- and post-test after working through the ASK ME-Algebra course.

Activity Information

Date Saved	Location	Title
Aug 25, 2004	Unit 1: The Speeding Problem	Activity 1: Describing Fines with Algebra
Oct 27, 2004	Unit 1: The Speeding Problem	Activity 2: Using Independent and Dependent Variables to Make Predictions
Oct 27, 2004	Unit 1: The Speeding Problem	Activity 3: Domain and Range
Sep 20, 2004	Unit 1: The Speeding Problem	Activity 4: Using Functions to Make Predictions
Sep 21, 2004	Unit 1: The Speeding Problem	Activity 5: Graphing Function Data
Nov 04, 2004	Unit 1: The Speeding Problem	Activity 6: Graphs of Functions and the 2-Second Rule
Dec 02, 2004	Unit 1: The Speeding Problem	Activity 7: Function Graphs and Speeding Drivers
Sep 30, 2004	Unit 1: The Speeding Problem	Activity 8: Functions and Insurance Rates
Oct 01, 2004	Unit 1: The Speeding Problem	Activity 9: Using tables to think about speeding fines
Jan 20, 2005	Unit 1: The Speeding Problem	Activity 10: Analyzing speeding fine structures in other communities
Oct 18, 2004	Unit 1: The Speeding Problem	Activity 11: Interpreting your graphs and your data
Jan 26, 2005	Unit 2: The Trash Problem	Activity 1: Reviewing the Four-Corner Model
	Unit 2: The Trash Problem	Activity 2: Defining Linear Functions
Jan 21, 2005	Unit 2: The Trash Problem	Activity 3: Introduction to Motion

Figure 39: The first part of Vince P.'s saved work.

	Unit 2: The Trash Problem	Activity 4: Rate of Change I
	Unit 2: The Trash Problem	Activity 5: Rate of Change II
May 10, 2005	Unit 2: The Trash Problem	Activity 6: The Linear Parent Function
	Unit 2: The Trash Problem	Activity 7: A Variation of the Linear Parent Function: y = mx
	Unit 2: The Trash Problem	Activity 8: Another Variation of the Linear Parent Function: y = mx + b
	Unit 2: The Trash Problem	Activity 9: Writing Rules Given the Slope and y-intercept
	Unit 2: The Trash Problem	Activity 10: Writing Rules Given Points on the Line
	Unit 2: The Trash Problem	Activity 11: Using Function Notation
	Unit 2: The Trash Problem	Activity 12: Practicing Writing Rules
	Unit 2: The Trash Problem	Activity 13: Practicing Linear and Non-Linear Functions
Feb 04, 2005	Unit 3: The Animal Shelter Problem	Activity 1: Exploring Functions and Equations
	Unit 3: The Animal Shelter Problem	Activity 2: Looking Closer at Equations

Figure 40: The second part of Vince P.'s work.

Vince saved all of 11 activities in Unit 1 but he only saved 3 activities in Unit 2 and 1 activity in Unit 3. Vince's grades for the Graded Assignments are:

Table 13: Vince's grades (percent correct)

Grades and Constructs	Unit 1	Unit 2	Unit 3
Variables	U1GA2, 86%, U1GA3, 85% and U1GA4, 100%	U2GA1, 85%, U2GA2, 100% and U2GA3, 94%	
Expressions		U2GA4, 85% and U2GA5, 81%	U3GA2, 90%
Equality			U3GA1, 85%, U3GA2, 90% and U3GA4, 96%
Other	U1GA1, 100% and U1GA5, 88%	U2GA6, 67%	U3GA3, 100% and U3GA5, 74%

These data seem to suggest that Vince understands variables, expressions and equality as assessed by the Graded Assignments.

During the first interview, Vince admitted that his previous mathematics class grades were "barely passing, in the 70's." Vince admitted that he does use algebra in his science class. Vince stated his father likes mathematics, and would help Vince with his mathematics homework, *if* Vince brought it home. Both of Vince's parents finished high school. Vince plans to go to a technical school or college "where I could, like, mess with computers and stuff like that."

He was asked to find the value in the box of $8 + 4 = \square + 5$. He asked what the box meant and the interviewer said it was similar to a variable. He replied, "Dang. I can't look up variable or nothing like that?" *No. But I'll tell you: A variable is something that represents different numbers.* "It has to be a number that goes in here?" *Right.* (Pauses) "And a plus sign. Where's that …" *Where is what?* "So, this has to be my answer right here?" *Yes.* "I think I have seen this one. Did you, like, make this up?"

Yes. (Pauses) "This is a number. (Pauses) I don't know. I know 12 doesn't go there cause your not adding these two right?" *It is whatever you decide*. "I don't want to do this." *Okay, I can give you the next one*.

He was asked if x – y = z – y is always true, never true, or sometimes true? He responded, "If sometimes, when? That is on the computer a lot, too. Gah, I hate when I… (Pauses) Hmm, I don't know." His response to simplify 3a – (b + a) was, "That is also on the computer. This is what I hate, there are no numbers." *If you want to use numbers, you can*. "I can't do this."

These data seem to suggest that Vince does not understand variables, expressions or equality. He did not answer any task given to him. He could instead have very low self-confidence in his mathematical ability. He wanted the interviewer to validate his response before deciding that 12 was or was not the answer.

By May 25, 2005, Vince had accumulated 1416 points out of 1600 for an 89%. Vince took the TAKS test during April 2005 and did not pass the mathematics portion. He is signed up take the test during April 2006.

Overall, Vince has a better understanding of variables, expressions and equality than he did before taking the ASK ME-Algebra course since his post-test score was perfect and his GA scores were 80% or higher.

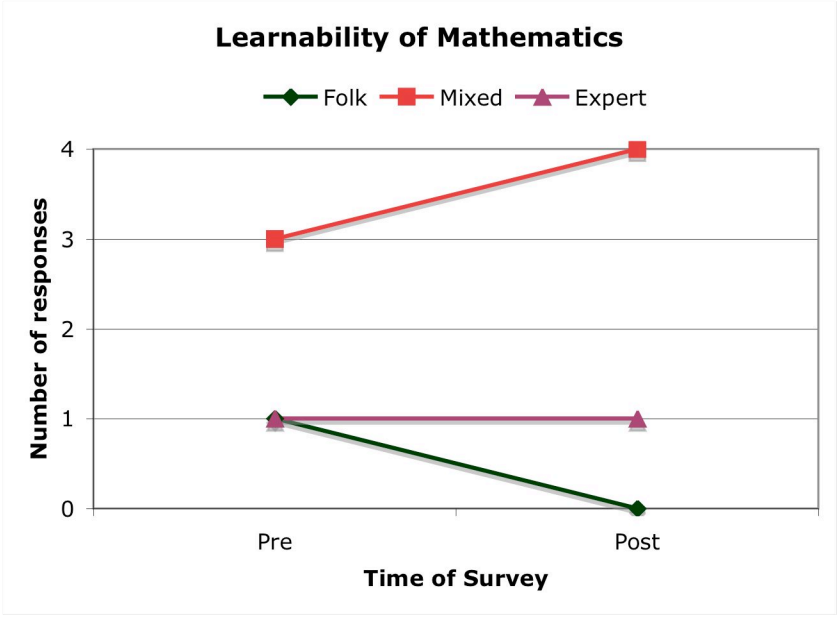


Figure 41: Vince's views of the "Learnability" of Mathematics

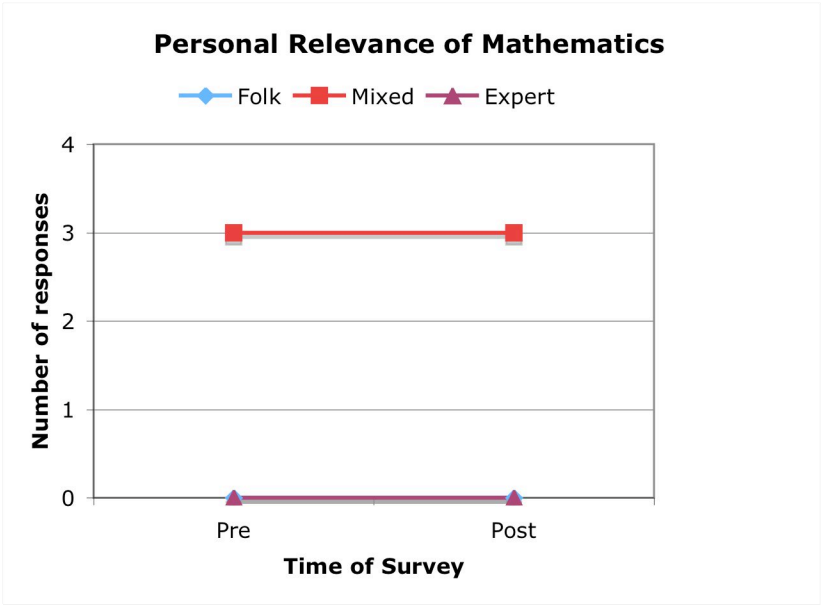


Figure 42: Vince's views of the Personal Relevance of Mathematics

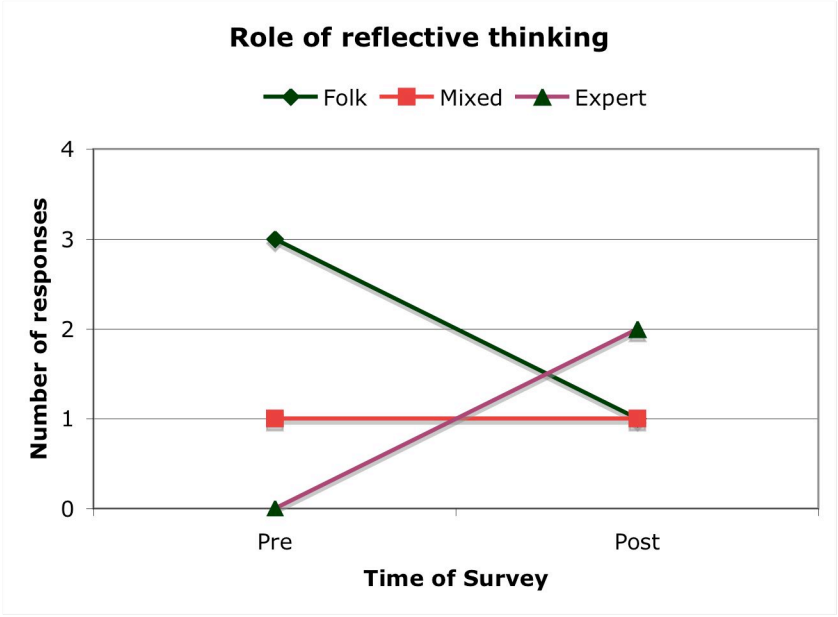


Figure 43: Vince's views of the Role of Reflective Thinking

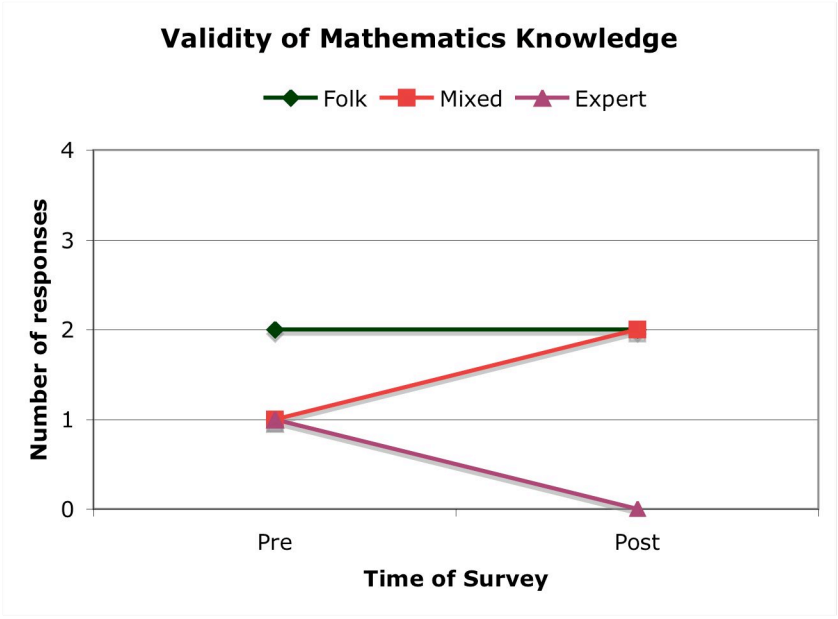


Figure 44: Vince's views of the Validity of Mathematics Knowledge

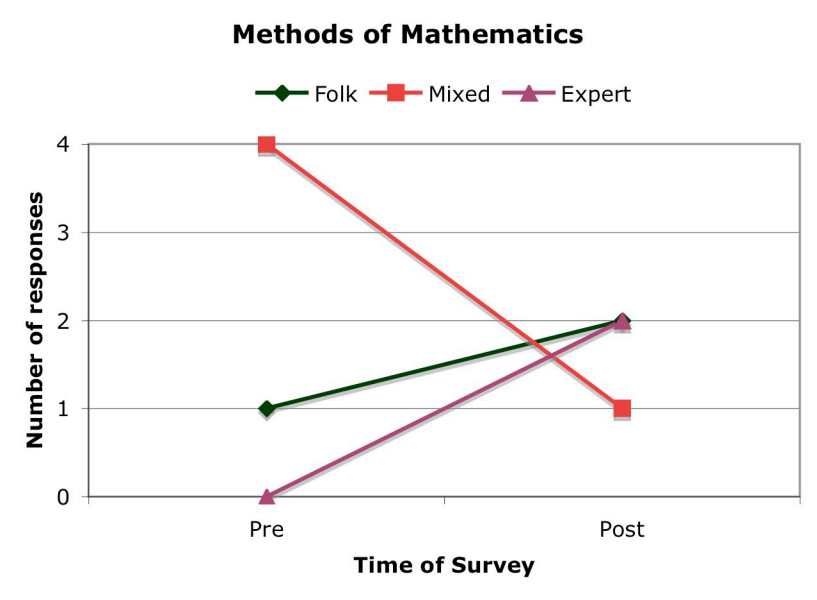


Figure 45: Vince's views of the Methods of Mathematics

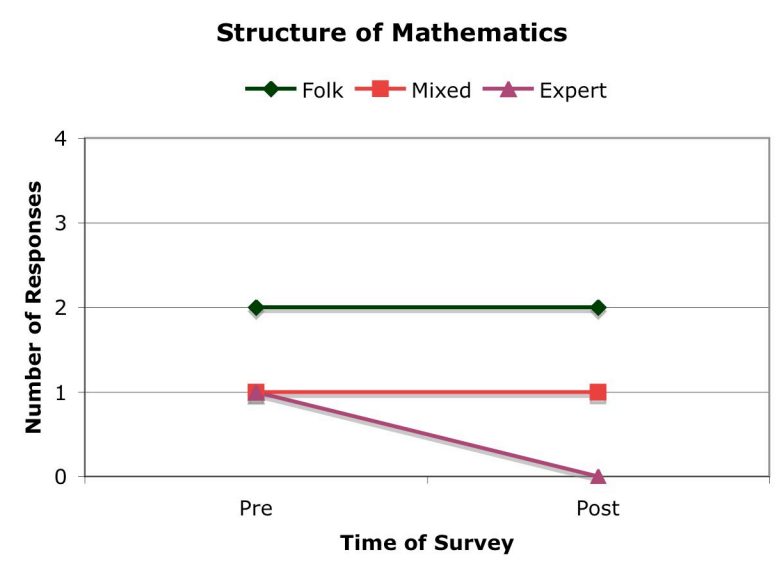


Figure 46: Vince's views of the Structure of Mathematics

Vince's views about mathematics do not seem to change overall. Many of his views stayed the same: Personal Relevance, Validity, and Structure. Vince had a

negative view of the course during the first interview and his view seemed unchanged during the final interview.

Interpretation

Six of the eight students have a better understanding of the concepts of variables, expressions, and equality after working through the GA that assessed the three algebraic concepts in the ASK ME-Algebra course. The main reason for this statement is that the six students performed well on the post-test and on the GA. The poor performance of the student on the interview tasks makes this researcher question the reliability and validity of the tasks. Maybe the four students that have not passed the TAKS test will score well on the April 2006 administration of the TAKS test and those results will bare the above conclusion out. Another possibility is that the TAKS test and GA are invalid by the teacher's intervention – implying that the interview tasks are valid and the students did not gain a complete understanding of variables, expressions, and equality.

A look at the three concepts across the students is next. The three concepts that are used are from the first research question of this study. That question is: In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' understanding of concepts, such as variables, equality, and equivalent expressions?

Students' understanding of variables

The second interview question that attempted to determine if the treatment group understood variables was, "Is x – y = z – y true always, never, or sometimes? If sometimes, when?" All but one student guessed, "Never." Vince stated, "I don't know."

The following TAKS items addressed students' understanding of variables:

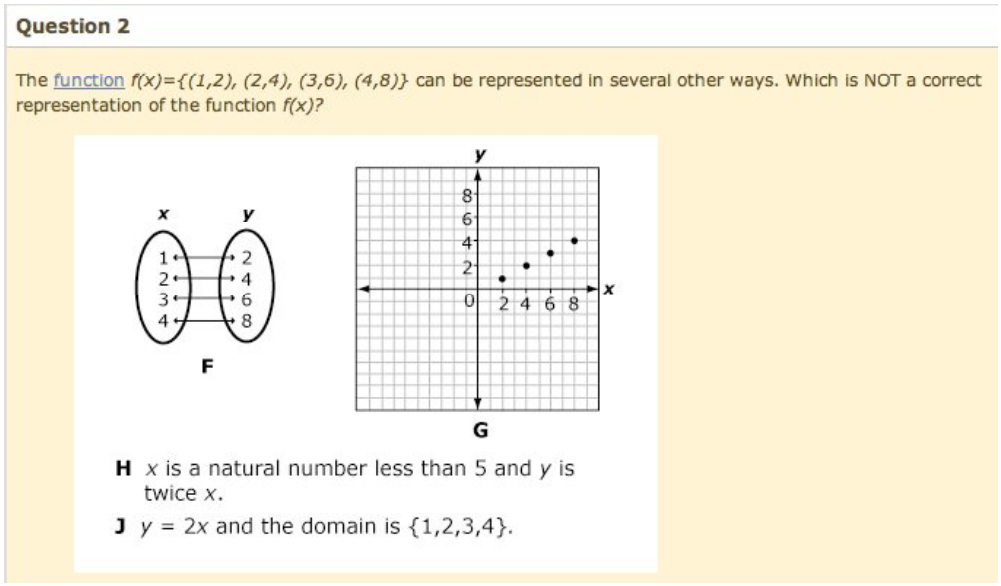


Figure 47: Question 2 from the Pre-Test of TAKS released items

Question 2 was on the pre-test and was in Unit 1 Graded Assignment 1 as Question 8. All of the students responded correctly to Question 8 on the post-test by choosing G.

Question 4

The equation $c = 0.75t$ represents c the total cost of t tickets on a bus. Which table contains values that fit this equation?

F

Cost of Bus Tickets				
t	1	2	3	4
c	$.75	$1.50	$2.25	$3.00

G

Cost of Bus Tickets				
t	1	2	3	4
c	$.75	$1.00	$1.25	$1.50

H

Cost of Bus Tickets				
t	1	2	3	4
c	$1.75	$2.50	$3.25	$4.00

J

Cost of Bus Tickets				
t	1	2	3	4
c	$1.75	$2.75	$3.75	$4.75

Figure 48: Question 4 from the pre-test of TAKS released items

Question 4 was on the pre-test and was in Unit 1 Graded Assignment 2 as Question 5. All but one student chose the correct response for Question 5 on the post-test; Mary chose H.

Question 8

Students in a science class recorded lengths of a stretched spring, as shown in the table below.

Length of Stretched Spring

Distance Stretched, x (centimeters)	Weight, y (newtons)
0	0
2	10
4	20
7	35
9	45
10	50

Which equation best represents the relationship between distance stretched x and the weight on the spring y?

F $y = -5x$

G $\frac{5}{x}$

H $y = 5x^2$

J $y = 5x$

Figure 49: Question 8 from the Pre-Test of TAKS released items

Question 8 was on the pre-test and was in Unit 1 Graded Assignment 3 as Question 10 on the post-test. Ana and Nan made incorrect choices for Question 10.

Question 9

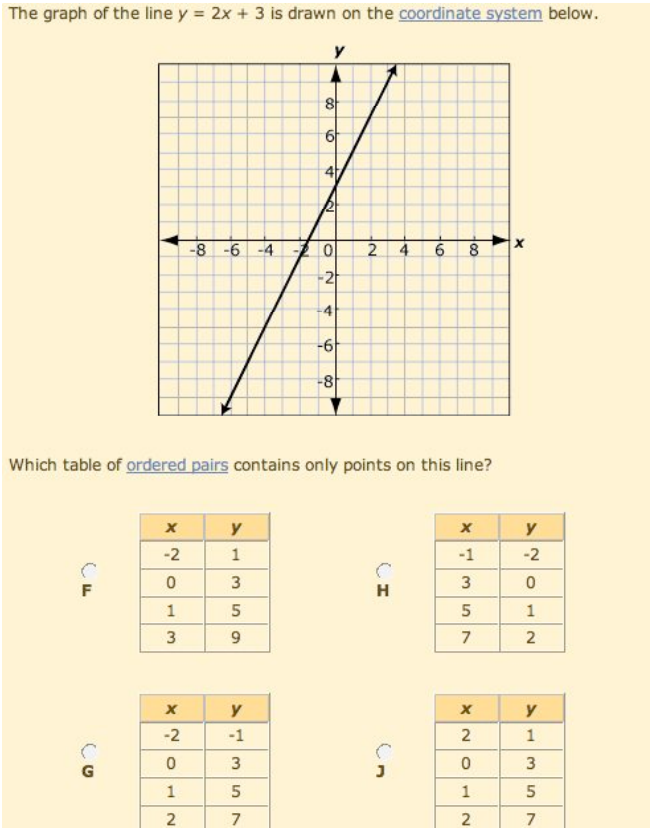
Which equation best describes the relationship between the corresponding values of x and y shown in the table?

x	y
−2	−12
0	−6
1	−3
4	6

- **A** $y=x-10$
- **B** $y=2x-8$
- **C** $y=3x-6$
- **D** $y=x^2-8$

Figure 50: Question 9 from the Pre-Test of TAKS released items

Question 9 was on the pre-test and was in Unit 1 Graded Assignment 3 as Question 9 on the post-test. Ana was the only student to choose an incorrect response for Question 9.



x	y
-2	1
0	3
1	5
3	9

x	y
-1	-2
3	0
5	1
7	2

x	y
-2	-1
0	3
1	5
2	7

x	y
2	1
0	3
1	5
2	7

Figure 51: Question 12 from the Pre-Test of TAKS released items

Question 12 was on the pre-test and was in Unit 1 Graded Assignment 4 as Question 5 on the post-test. All of the students made the correct choice for Question 5.

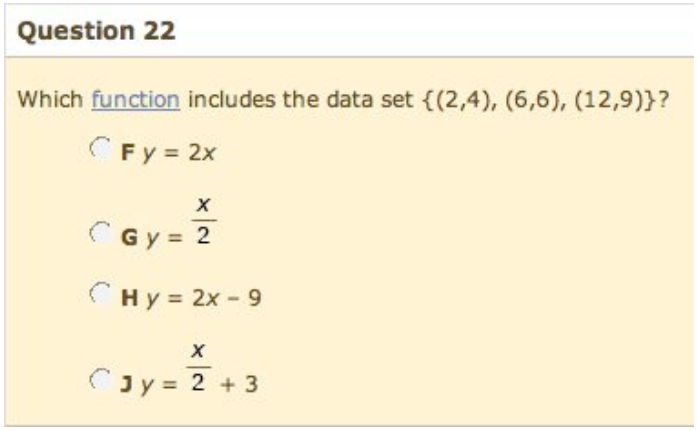


Figure 52: Question 22 from the Pre-Test of TAKS released items

Question 22 was on the pre-test and was in Unit 2 Graded Assignment 4 as Question 9 on the post-test. Ana was the only student who did not receive credit for Question 9.

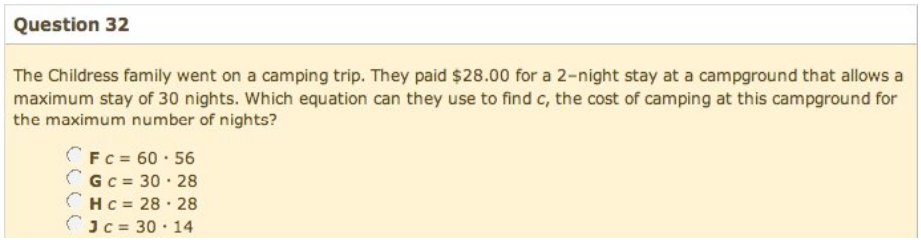
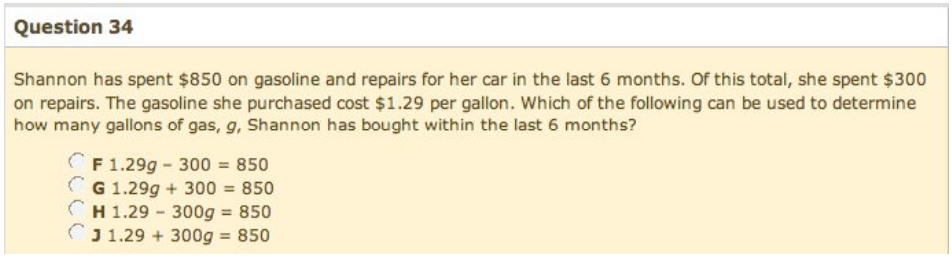


Figure 53: Question 32 from the Pre-Test of TAKS released items

Question 32 was on the pre-test and was in Unit 3 Graded Assignment 3 as Question 9 on the post-test. Mary was the only student who answered Question 9 incorrectly.



Question 34

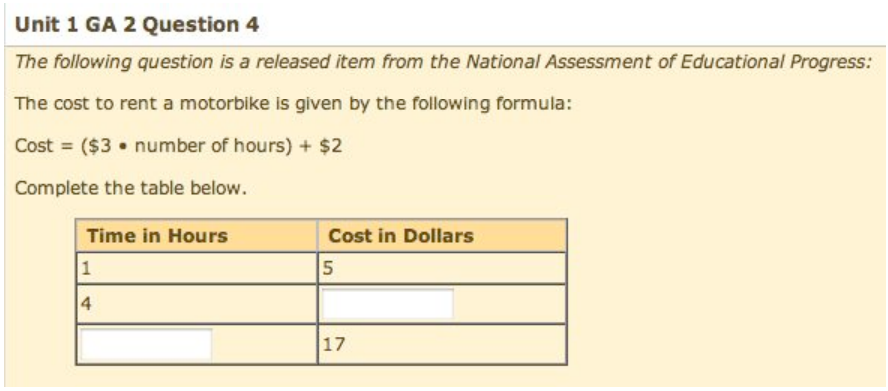
Shannon has spent $850 on gasoline and repairs for her car in the last 6 months. Of this total, she spent $300 on repairs. The gasoline she purchased cost $1.29 per gallon. Which of the following can be used to determine how many gallons of gas, *g*, Shannon has bought within the last 6 months?

- **F** $1.29g - 300 = 850$
- **G** $1.29g + 300 = 850$
- **H** $1.29 - 300g = 850$
- **J** $1.29 + 300g = 850$

Figure 54: Question 34 from the Pre-Test of TAKS released items

Question 34 was on the pre-test and was in Unit 3 Graded Assignment 4 as Question 8. Mary was the only student who answered Question 8 incorrectly.

The following were questions from the Graded Assignments that attempt to determine if students understood the concept of variables:



Unit 1 GA 2 Question 4

The following question is a released item from the National Assessment of Educational Progress:

The cost to rent a motorbike is given by the following formula:

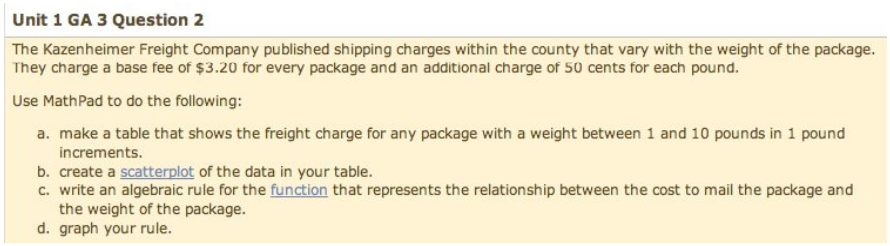
Cost = ($3 • number of hours) + $2

Complete the table below.

Time in Hours	Cost in Dollars
1	5
4	
	17

Figure 55: Unit 1 Graded Assignment 2 Question 4

All of the students answered Unit 1 Graded Assignment 2 Question 4 correctly.



Unit 1 GA 3 Question 2

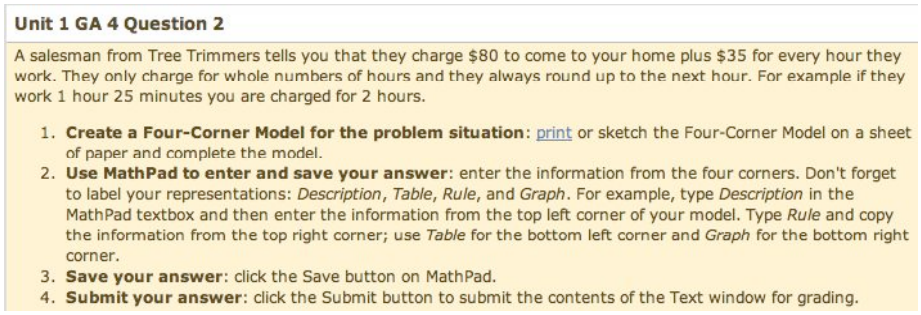
The Kazenheimer Freight Company published shipping charges within the county that vary with the weight of the package. They charge a base fee of $3.20 for every package and an additional charge of 50 cents for each pound.

Use MathPad to do the following:

a. make a table that shows the freight charge for any package with a weight between 1 and 10 pounds in 1 pound increments.
b. create a scatterplot of the data in your table.
c. write an algebraic rule for the function that represents the relationship between the cost to mail the package and the weight of the package.
d. graph your rule.

Figure 56: Unit 1 Graded Assignment 3 Question 2

Ana, Barbara, Jean, Mary answered Unit 1 Graded Assignment 3 Question 2 correctly[36]. Bubba did not include a scatterplot of the table of values with his response. Günter, Nan, and Vince did not include a table of values with theirs.



Unit 1 GA 4 Question 2

A salesman from Tree Trimmers tells you that they charge $80 to come to your home plus $35 for every hour they work. They only charge for whole numbers of hours and they always round up to the next hour. For example if they work 1 hour 25 minutes you are charged for 2 hours.

1. **Create a Four-Corner Model for the problem situation**: print or sketch the Four-Corner Model on a sheet of paper and complete the model.
2. **Use MathPad to enter and save your answer**: enter the information from the four corners. Don't forget to label your representations: *Description*, *Table*, *Rule*, and *Graph*. For example, type *Description* in the MathPad textbox and then enter the information from the top left corner of your model. Type *Rule* and copy the information from the top right corner; use *Table* for the bottom left corner and *Graph* for the bottom right corner.
3. **Save your answer**: click the Save button on MathPad.
4. **Submit your answer**: click the Submit button to submit the contents of the Text window for grading.

Figure 57: Unit 1 Graded Assignment 4 Question 2

Ana, Bubba, Barbara, Günter, and Vince answered Unit 1 Graded Assignment 4 Question 2 correctly. Jean did not include a table of values with her response. Mary did not include a verbal description or a rule with hers. Nan included everything but labeled the verbal description as the algebraic rule in her response.

[36] The following evaluations of the students' GA are mine not the teacher's from Gaggle HS.

Unit 2 GA 1 Question 1

Red wiggler worms cost about $20 per pound when you buy them in large quantities.
Source: Cascade Sales: The Worm Factory

Use the Math Tool below to create the four representations in a Four-Corner Model:

- make a table of values that represents this situation,
- use this table to make a scatterplot,
- write an algebraic rule, and
- graph the algebraic rule.

Figure 58: Unit 2 Graded Assignment 1 Question 1

All of the students, except one, answered correctly to Unit 2 Graded Assignment 1 Question 1. Vince did not include a graph of the algebraic rule with his response.

Unit 2 GA 2 Question 5

Below is a table of values representing certain points on one trip the garbage truck made, showing the truck's distance from the motion detector at the given times.

Time (in seconds)	Distance (in feet)
0	6
1	4.5
2	3
3	1.5
4	0
t	***d***

Use the MathPad to do the following:

a. Write a description of the trip in words. (Enter your answer in the textbox portion of the MathPad below.)
b. Graph the trip.
c. Write an algebraic rule for the distance, *d*, of this trip from the motion detector for any number of seconds, *t*. (Enter your answer in the textbox portion of the MathPad below.)

Figure 59: Unit 2 Graded Assignment 2 Question 5

Ana, Mary, Nan, and Vince were correct on Unit 2 Graded Assignment 2 Question 5. Bubba did not have the correct algebraic rule for the truck's motion in his response. Barbara, Günter, and Jean did not have a complete description of the truck's motion in theirs.

Figure 60: Unit 2 Graded Assignment 3 Question 5

All of the students correctly answered Unit 2 Graded Assignment 3 Question 5.

Interpreting Results of Variables

Table 14: Summary of Students' Understanding of Variables

	Interview Task	**6 Graded Assignment Problems**	**8 Post-test Questions**
Least Correct	7 incorrect responses	4 students correctly answered U1GA3 Q2 and U2GA2 Q5	6 students correctly answered question 8
	1 partially correct answer	6 students correctly answered U2GA1 Q1 5 students correctly answered U1GA4 Q2	7 students correctly answered questions 4, 9, 22, 32, 34
Most Correct		All students correctly answered U1GA2 Q4 and U2GA3 Q5	All students correctly answered questions 2 and 12

Here is my interpretation of the results on the students' understanding of variables. None of the students correctly answered the interview question that evaluated their understanding of variables. Of the 8 post-test items that assessed variables, Mary and Ana incorrectly answered 3 of them; Nan incorrectly answered 1 of them; Jean, Bubba, Barbara, Günter, and Vince answered all of them correctly. Of the 6 Graded Assignment problems that assessed variables, Jean answered all of them correctly; Mary, Nan, and Barbara incorrectly answered 1 of them; Bubba, Günter, and Vince incorrectly answered 2 of problems.

The interview question seemed as if it was not an appropriate question for this group of students since the students were unfamiliar with non-contextual problems. Overall, the data does seem to be positive and the students did show an improvement in

their understanding of variables as assessed by comparing the answers given for the pre-test versus the post-test questions and as assessed by the GA problems.

Some of the students were able to discuss the meaning of "independent variable" and "dependent variable" during the second interview but none of them could simplify the task in the interview, therefore, the students are still "process oriented" and did not developed variables into "procepts" (Graham & Thomas, 2000). This was evidenced in the second interview of the students who attempted to simplify 3a – (b + a): Bubba wrote "b+3a", Barbara stated "3a+b", Günter wrote, "3a – 3b + -3a" after he was asked to combine like terms (he did distribute the negative correctly), Jean tried to solve the equation even though there "was no equals". Nan wrote, $3a - b + 3a^2$. The processes each student used did not take into account the objects (a and b). Bubba and Barbara knew the "a's" needed to become one term but did not use the fact that "a" represented "1a". Günter knew how to distribute -1 but not how to combine like terms and Nan partially distributed the wrong value (3a instead of -1).

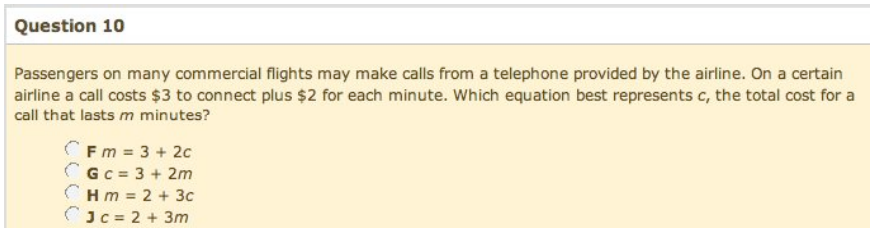
Two students understood the need to use the distributive property but only one student was able to distribute -1. The students can do the process and yet not understand what the objects being worked with are representing.

One positive note is that the five students who did work the problem did not feel the need to reduce their response to a single term, so the students had advanced past using arithmetic until there was a single term (Wagner & Parker, 1993).

Students' understanding of expressions

The interview question that attempted to determine if the treatment group understood the concept of expressions was the problem, "Simplify 3a – (b + a)." None of the students found the correct answer, 2a – b. Many did not even try to work through the problem. Günter's answer was partially correct, since he distributed the negative sign correctly but he did not combine like terms correctly.

The TAKS post-test items that addresses students' understanding of expressions were the following:



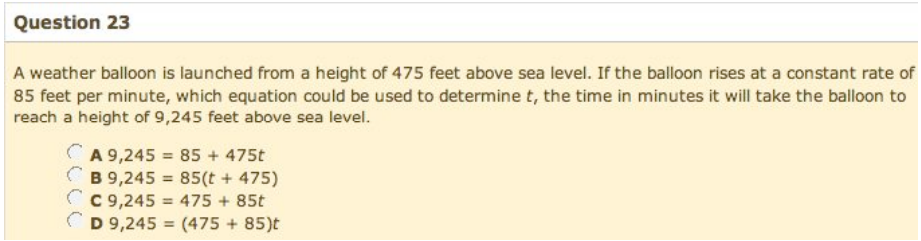
Question 10

Passengers on many commercial flights may make calls from a telephone provided by the airline. On a certain airline a call costs \$3 to connect plus \$2 for each minute. Which equation best represents c, the total cost for a call that lasts m minutes?

- **F** $m = 3 + 2c$
- **G** $c = 3 + 2m$
- **H** $m = 2 + 3c$
- **J** $c = 2 + 3m$

Figure 61: Question 10 from the Pre-Test of TAKS released items

Question 10 was on the pre-test and was in Unit 1 Graded Assignment 4 as Question 3 on the post-test. All of the students in the treatment group answered Question 3 correctly.



Question 23

A weather balloon is launched from a height of 475 feet above sea level. If the balloon rises at a constant rate of 85 feet per minute, which equation could be used to determine t, the time in minutes it will take the balloon to reach a height of 9,245 feet above sea level.

- **A** $9{,}245 = 85 + 475t$
- **B** $9{,}245 = 85(t + 475)$
- **C** $9{,}245 = 475 + 85t$
- **D** $9{,}245 = (475 + 85)t$

Figure 62: Question 23 from the Pre-Test of TAKS released items

Question 23 was on the pre-test and was in Unit 2 Graded Assignment 4 as Question 10 on the post-test. Ana was the only student to answer Question 10 incorrectly.

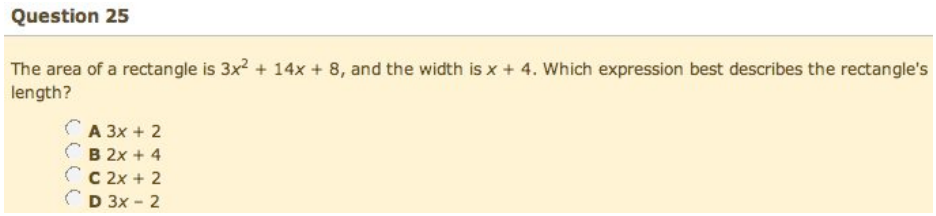


Figure 63: Question 25 from the Pre-Test of TAKS released items

Question 25 was in the pre-test and was in Unit 3 Graded Assignment 2 as Question 7 on the post-test. Barbara, Günter, and Nan did not respond to Question 7 correctly.

Question 26

Simplify the algebraic expression $5(x + 3)(x + 2) - 3(x^2 + 2x + 1)$.

- **F** $2x^2 + 7$
- **G** $2x^2 + 27$
- **H** $2x^2 + 7x + 7$
- **J** $2x^2 + 19x + 27$

Figure 64: Question 26 from the Pre-Test of TAKS released items

Question 26 was on the pre-test and was in Unit 3 Graded Assignment 2 as Question 8 on the post-test. Nan was the only student to answer Question 8 incorrectly.

Question 27

Simplify the algebraic expression $3(x + 3) - 2(x + 3)$.

- **A** $x + 3$
- **B** $x - 3$
- **C** $-6x^2 - 54$
- **D** $6x^2 + 3$

Figure 65: Question 27 from the Pre-Test of TAKS released items

Question 27 was on the pre-test and was in Unit 3 Graded Assignment 2 as Question 9 on the post-test. Barbara and Günter answered Question 9 incorrectly.

The following were the questions in the Graded Assignments that address the concept of expressions:

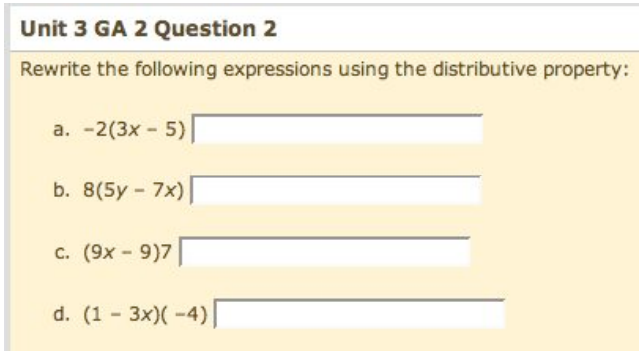


Figure 66: Unit 3 Graded Assignment 2 Question 2

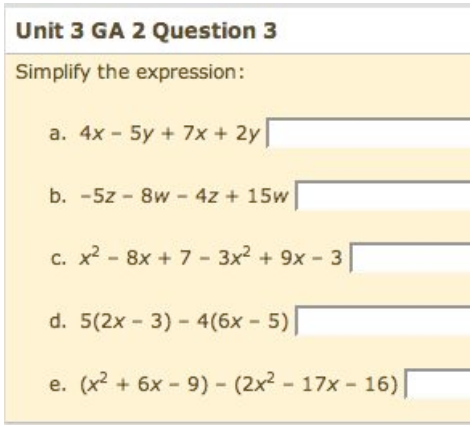


Figure 67: Unit 3 Graded Assignment 2 Question 3

Ana, Günter, Nan, Victor, and Brandon's answers to Unit 3 Graded Assignment 2 Question 2 and Question 3 were alike since their wrong answers were the same. Here is a sample answer:

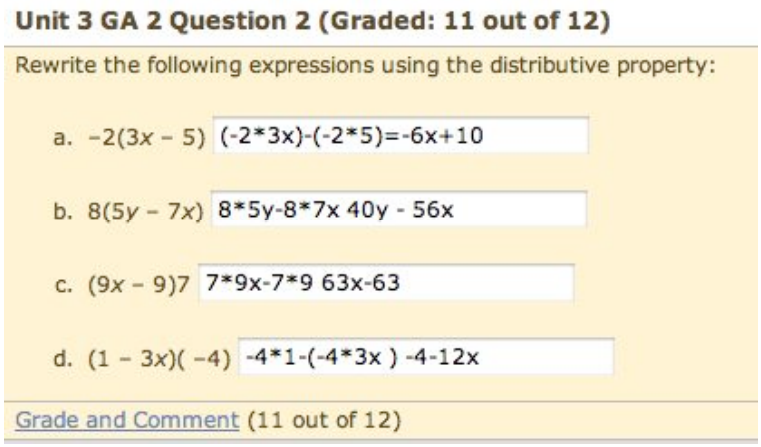


Figure 68: Sample answers from Unit 3 Graded Assignment 2 Question 2

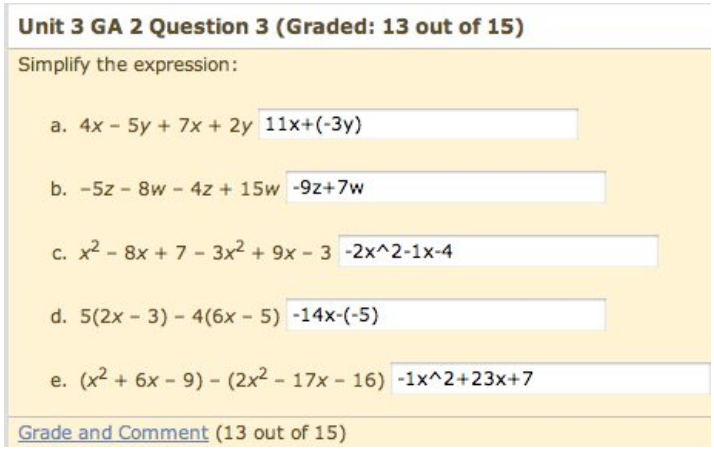


Figure 69: Sample answers from Unit 3 Graded Assignment 2 Question 3

Mary was correct on Question 2 part “d” but she has a typo in part “e” of Question 3 and she had the same incorrect response for part “c”, as above. Barbara and Jean’s responses to part “d” of Question 2 and part “c” of Question 3 were correct.

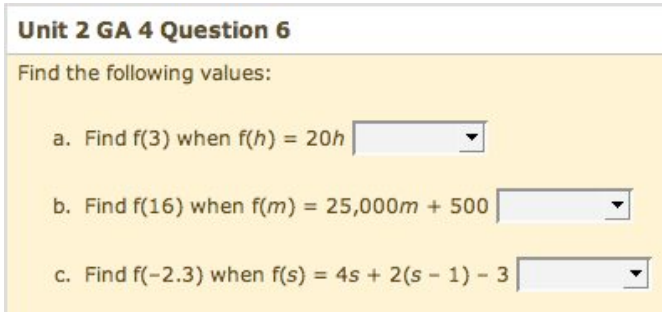


Figure 70: Unit 2 Graded Assignment 4 Question 6

All of the students responded to Unit 2 Graded Assignment 4 Question 6 with 60, 400500, and –18.8. Overall, only 2 students answered the interview question correctly. Of the 5 post-test TAKS items, Barbara, Günter and Nan answered two incorrectly. Ana answered 1 incorrectly. Of the 3 Graded Assignment problems that assessed expressions, 6 of the 8 students seemed to have copied the incorrect responses to parts of two problems from one another.

Interpreting Results of Expressions

Table 15: Summary of Students' Understanding of Expressions

	Interview Task	**3 Graded Assignment Problems**	**5 Post-test Questions**
Least correct	All incorrect responses	5 students copied answers to U3GA2 Q2 and U3GA2 Q3	5 students correctly answered question 25
		3 students partially answered U3GA2 Q2 and U3GA2 Q3	6 students correctly answered question 27
Most Correct			7 students correctly answered questions 23 and 26 All students correctly answered question 10

Of the three concepts covered in this study, simplifying expressions was the least emphasized by the ASK ME-Algebra course (as evidenced by the few GA questions). The data seem to indicate that the students had a difficult time understanding expressions and finding equivalent expressions. Overall, the students did not improve their understanding of expressions while using ASK ME-Algebra from their level of understanding before taking the course.

The concept of expressions was assessed during the time that the teacher was helping the students. It is possible that the teacher allowed the students to not save their work in the activities, which is where the bulk of the curriculum is located.

The students did not have a complete understanding of what they were doing with the expressions. They were able to "manipulate symbols" as evidenced by their work in U3GA2 Q2 and Q3 but since they did not understand what "the mathematical objects"

represented the students were unable to detect the incorrect responses (Kieran & Sfard, 1999, p. 2).

Students' understanding of Equality

Research has shown that a relational understanding of equality, as opposed to an operational understanding of equality, is needed for success in algebra and future mathematics classes. The interview question that attempted to determine if the treatment group had a relational understanding of equality was, "Find the value that goes in the box and think aloud. $8 + 4 = \square + 5$."

Two students were able to give the correct response of "7". Günter solved the problem by subtracting 4 from both sides and then seeing that 8 equals something plus one, while Jean said she added the left side of the equation together and then tried to figure out what needed to be with 5 to make the right side the same value.

Four students (Ana, Bubba, Barbara and Nan) stated that the answer was 12. Each student's reason was that he or she had added 8 and 4. Mary said the answer was 17 since she had added 8, 4, and 5 together. Vince stated that he did not want to try to answer the problem after asking, "It's not 12, is it?" The interviewer responded that it was whatever he decided.

The TAKS items that addressed equality were the following:

Question 29

Which problem situation matches the equation below?

$x - 4.72 = 5.28$

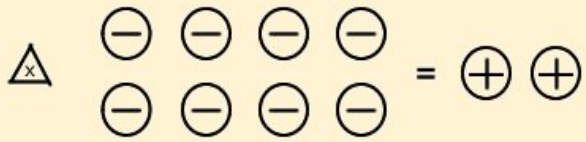
- **A** Sergio's lunch cost $4.72. He receives $5.28 in change when he paid the bill. What is x, the amount of money he gave the cashier?
- **B** Yvette cycled 4.72 kilometers in a race. The winning cyclist's time was 5.28 seconds faster then Yvette's. What is x, the time in seconds it took Yvette to finish the race?
- **C** Janice and Maura measured the wingspans of butterflies in science class. Janice's butterfly had a wingspan of 4.72 centimeters, and Maura's butterfly had a wingspan 5.28 centimeters. What is x the average length of a butterfly's wingspan?
- **D** Mrs. Castro paid $4.72 for a jar of iced-tea mix that was originally priced at $5.28. What is x, the amont of money that Mrs. Castro saved altoghter?

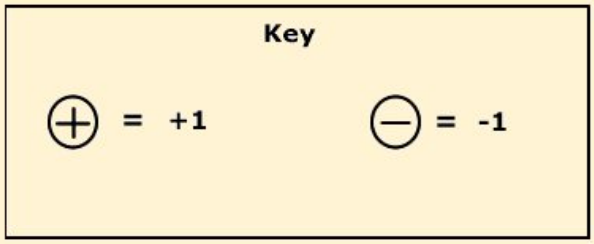
Figure 71: Question 29 from the pre-test of TAKS released items

Question 29 was on the pre-test and was in Unit 3 Graded Assignment 3 as Question 6 on the post-test. Mary and Nan did not respond correctly to Question 6.

Question 31

The model represents the equation





What is the value of x?

- **A** $x = -6$
- **B** $x = 4$
- **C** $x = 8$
- **D** $x = 10$

Figure 72: Question 31 from the pre-test of TAKS released items

Question 31 was on the pre-test and was in Unit 3 Graded Assignment 3 as Question 8 on the post-test. Mary and Nan did not respond correctly to Question 8.

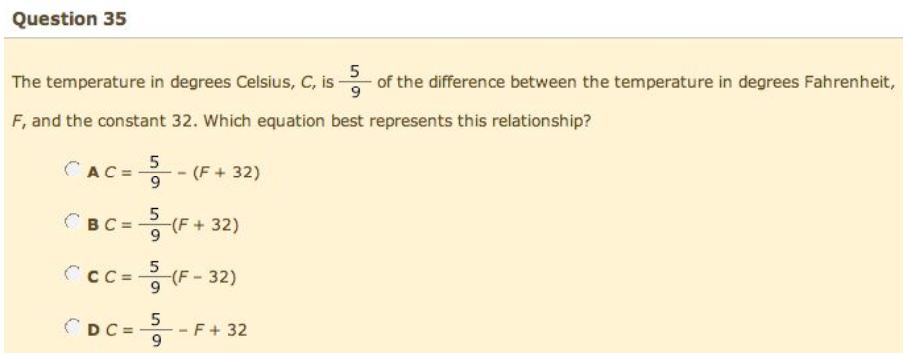


Figure 73: Question 35 from the pre-test of TAKS released items

Question 35 was on the pre-test and was in Unit 3 Graded Assignment 4 as Question 9 on the post-test. All of the students responded to Question 9 correctly.

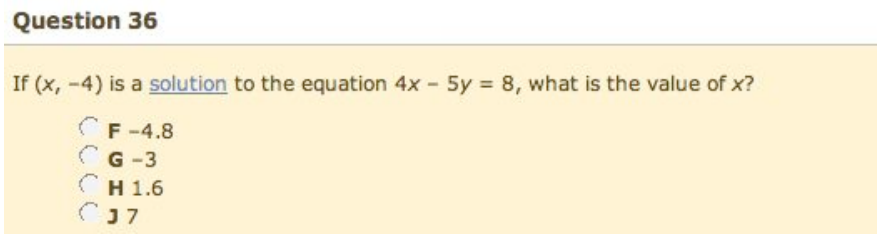


Figure 74: Question 36 from the pre-test of TAKS released item

Question 36 was on the pre-test and was in Unit 3 Graded Assignment 4 as Question 10 on the post-test. Nan did not choose the correct answer for Question 10.

The following were questions from the Graded assignments that test students' understanding of equality:

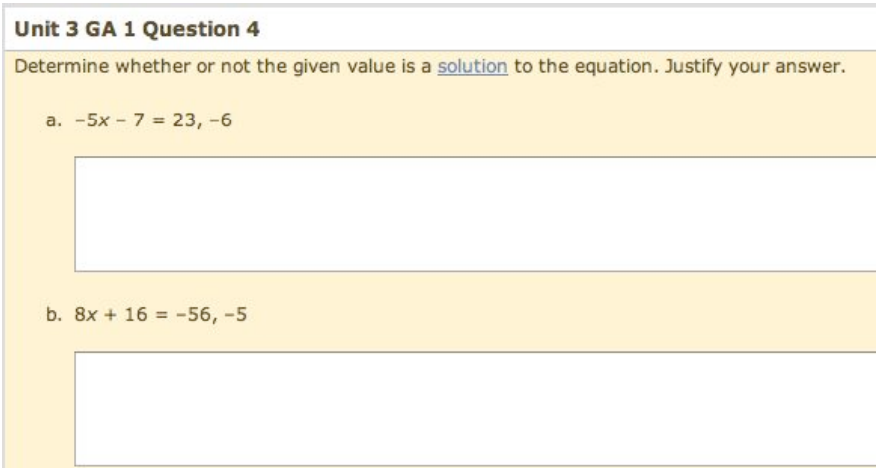


Figure 75: Unit 3 Graded Assignment 1 Question 4

All of the students said yes and no to Unit 3 Graded Assignment 1 Question 4. The justifications were very similar in that they all spelled *does*, "dose" and *doesn't*, "dosn't".

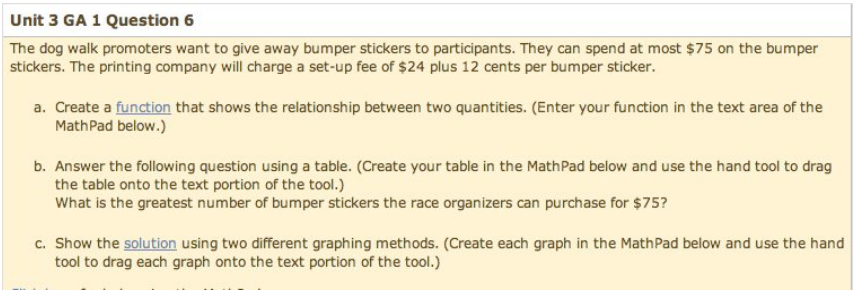


Figure 76: Unit 3 Graded Assignment 1 Question 6

Ana was completely correct on Unit 3 Graded Assignment 1 Question 6. Bubba, Barbara, Günter, Jean, Nan, and Vince used only one graphing method in their response. Mary did not graph the table of values correctly in hers.

Unit 3 GA 2 Question 4

Oliver's Obedience School has decided to give away t-shirts at the Bark in the Park Fair. They have $240 in their budget to spend on the shirts. The company that will make the shirts charges $32 to design the logo plus $3.52 per shirt. How many shirts can they buy for $240?
In the MathPad below, do the following:

- Write an equation to answer the question, and enter it in the text area of the MathPad.
- Use the symbolic method to solve the equation, and show your work in the text area of the MathPad.
- Using the Table tab in the MathPad, create a table to verify your solution to the equation. Once you have created the table, use the Hand tool to drag your finished table to the text area.

Figure 77: Unit 3 Graded Assignment 2 Question 4

Ana rounded off the values in her table to Unit 3 Graded Assignment 2 Question 4. Bubba, Barbara, Günter, Jean, Nan were correct with their response. Mary did not use the symbolic method to solve the equation in her response. Vince did not have an equation and did not use the symbolic method in his response.

Unit 3 GA 2 Question 5

At the Bark in the Park fair, the non-profit Greyhound Rescue organization will set up a booth. The following information appears on the vendor space application:

10' x 10' Booth Rental Charge (choose only one)	Cost	Equipment Charges	Cost
Vendor Space	$175	Table	$15
Non Profit group with Items for sale	$75	Chair	$3
Non Profit without items for sale	$35	Electricity	$50
_	_	Canopy	$150

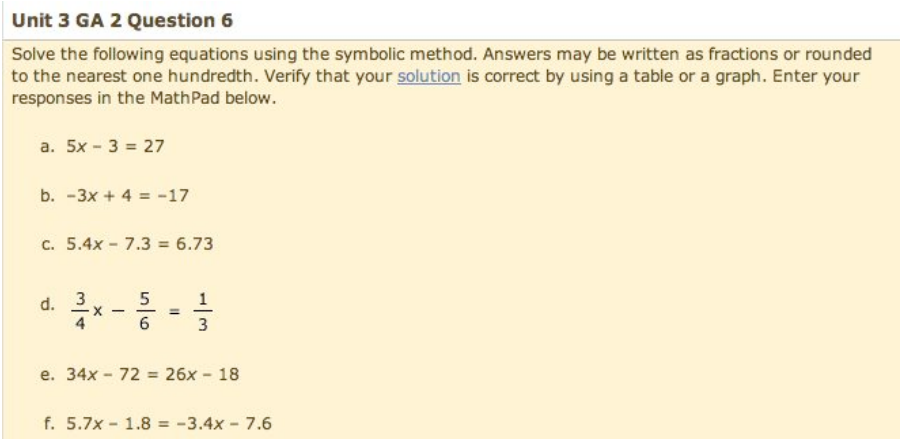
They select one 10' by 10' booth with 2 tables and 2 chairs. They will spend $48 on fliers to advertise their group. They have decided to sell dog tags and must rent a machine at a cost of $43 and the blank tags cost the group 57 cents each. The only income at the booth will come from the sale of the dog tags at $5 per tag. How many tags must they sell to break even?

Write an equation to answer the question. Use the symbolic method to solve the equation and verify your solution using a graph. Enter your response in the MathPad below.

Figure 78: Unit 3 Graded Assignment 2 Question 5

Ana, Bubba, Jean, and Vince were correct on Unit 3 Graded Assignment 2 Question 5. Barbara did not have the equation, symbolic method or graph in her response. Günter had an answer similar to Barbara's response. Mary included only the

equation in her response. Nan included the equation and graph but did not include the symbolic method or solution in hers.



Unit 3 GA 2 Question 6

Solve the following equations using the symbolic method. Answers may be written as fractions or rounded to the nearest one hundredth. Verify that your solution is correct by using a table or a graph. Enter your responses in the MathPad below.

a. $5x - 3 = 27$

b. $-3x + 4 = -17$

c. $5.4x - 7.3 = 6.73$

d. $\frac{3}{4}x - \frac{5}{6} = \frac{1}{3}$

e. $34x - 72 = 26x - 18$

f. $5.7x - 1.8 = -3.4x - 7.6$

Figure 79: Unit 3 Graded Assignment 2 Question 6

Ana and Bubba did not answer any of the problems on Unit 3 Graded Assignment 2 Question 6. Barbara and Jean had correct responses for parts a, b, and c, but no correct responses for parts d, e, and f. Günter has incorrect responses for parts e and f. Mary, Nan, and Vince have incorrect responses for parts b and c; they did not use the symbolic method to find their responses.

Problem Situation

For the Bark in the Park fair the following information appears on the vendor space application:

10' x 10' Booth Rental Charge	Cost	Equipment Charges	Cost
Vendor Space	$175	Table	$15
Non Profit group with Items for sale	$75	Chair	$3
Non Profit without items for sale	$35	Electricity	$50
_	_	Canopy	$150

There are 10,000 people expected to attend the event.
Use the above information to solve the following problems:

Unit 3 GA 4 Question 1

Gadget, Inc. plans to sell gadgets at the fair. Their expenses are the cost of the 10' x 10' booth for a vendor space, two tables, three chairs, a canopy and $5000 for the gadgets. The income will be from the charge of $7 per gadget and a donation of $500 from a local bank. How many gadgets must be sold to break even?

Figure 80: Unit 3 Graded Assignment 4 Question 1 and the problem situation for the question

Ana, Bubba, Jean, Nan, and Vince were correct on Unit 3 Graded Assignment 4 Question 1. Barbara and Günter wrote the equation with two equal signs in their response. Mary did not write an equation and did not find the correct value in her response.

Problem Situation

For the Bark in the Park fair the following information appears on the vendor space application:

10' x 10' Booth Rental Charge	Cost	Equipment Charges	Cost
Vendor Space	$175	Table	$15
Non Profit group with Items for sale	$75	Chair	$3
Non Profit without items for sale	$35	Electricity	$50
_	_	Canopy	$150

There are 10,000 people expected to attend the event.
Use the above information to solve the following problems:

Unit 3 GA 4 Question 2

The SPCA would like to set up a booth to hand out booklets about their organization. The expenses will be the cost of the booth of a non profit without items for sale, one table, two chairs and $0.45 for printing each booklet. Many donations were received and totaled $4154 to cover expenses for distributing the booklets. How many booklets can be printed?

Figure 81: Unit 3 Graded Assignment 4 Question 2 and the problem situation for the question

Ana and Nan did not find the correct equation or the correct answer to Unit 3 Graded Assignment 4 Question 2. Bubba and Vince wrote an equation with two variables and found an answer that was one number away from the correct answer in their response. Barbara, Günter, and Jean wrote the correct equation and found the correct answer in their response. Mary did not write an equation with symbols, she wrote in words the situation and the directions to find the answer in her response.

Problem Situation

For the Bark in the Park fair the following information appears on the vendor space application:

10' x 10' Booth Rental Charge	Cost	Equipment Charges	Cost
Vendor Space	$175	Table	$15
Non Profit group with Items for sale	$75	Chair	$3
Non Profit without items for sale	$35	Electricity	$50
_	_	Canopy	$150

There are 10,000 people expected to attend the event.
Use the above information to solve the following problems:

Unit 3 GA 4 Question 3

The Terrier Rescue Organization (TRO) is planning to sell dog identification tags. The total cost for manufacturing the tags and renting the booth is $200 and each tag will sell for $0.53. The volunteers will do the engraving for free at the fair. The TRO has a budget of $3000 for the booth and the dog identification tags. What is the minimum number of tags they should manufacture to break even?

Figure 83: Unit 3 Graded Assignment 4 Question 3 and the problem situation for the question

Ana, Bubba, Nan, and Vince wrote the correct function but they did not solve the specific equation; they approximated an answer to Unit 3 Graded Assignment 4 Question 3. Barbara, Günter, Jean, and Mary wrote the correct equation and correct answer in their response.

Unit 3 GA 4 Question 4

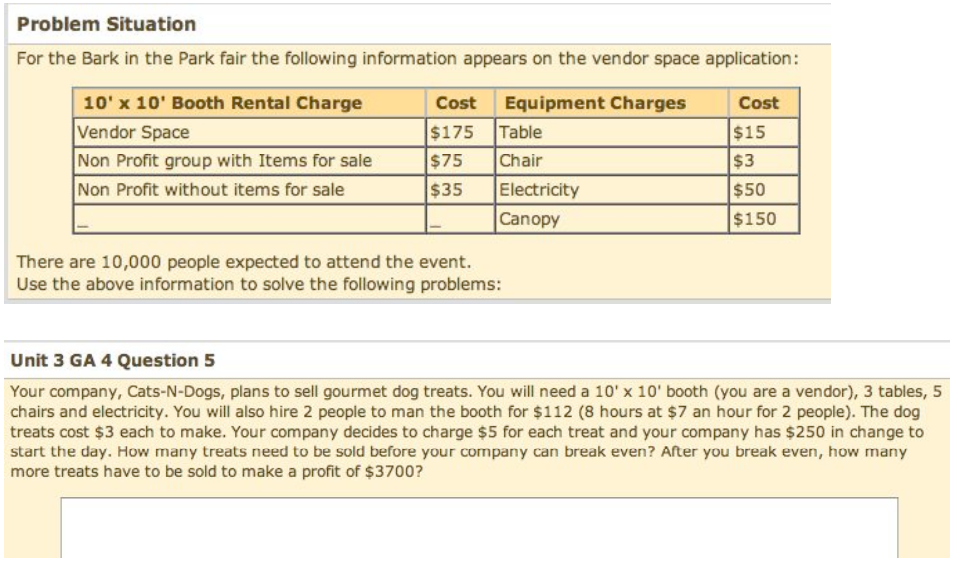
Solve the following. Make sure you show your work, or explain how you found the solution.

1. $3.06x = 1.57x + 378$
2. $2.48x = -4.5x + 604$
3. $78x = 56x - 5987$

Figure 84: Unit 3 Graded Assignment 4 Question 4

Ana, Barbara, Günter, Jean, Mary, and Nan wrote down just the answers to the three problems in Unit 3 Graded Assignment 4 Question 4. Bubba showed all of his

work but he did not find the correct answer for part 2 (he added "-4.5x" to both sides instead of adding "4.5x"). Vince showed his work but he was completely wrong for part 2 and he rounded all of his answers off to the nearest whole number in his response.



Problem Situation

For the Bark in the Park fair the following information appears on the vendor space application:

10' x 10' Booth Rental Charge	Cost	Equipment Charges	Cost
Vendor Space	$175	Table	$15
Non Profit group with Items for sale	$75	Chair	$3
Non Profit without items for sale	$35	Electricity	$50
_	_	Canopy	$150

There are 10,000 people expected to attend the event.
Use the above information to solve the following problems:

Unit 3 GA 4 Question 5

Your company, Cats-N-Dogs, plans to sell gourmet dog treats. You will need a 10' x 10' booth (you are a vendor), 3 tables, 5 chairs and electricity. You will also hire 2 people to man the booth for $112 (8 hours at $7 an hour for 2 people). The dog treats cost $3 each to make. Your company decides to charge $5 for each treat and your company has $250 in change to start the day. How many treats need to be sold before your company can break even? After you break even, how many more treats have to be sold to make a profit of $3700?

Figure 85: Unit 3 Graded Assignment 4 Question 5 and the problem situation for the question

Ana did not write an equation for breaking even but she did find the correct number of dog treats. She did not write an equation for making a profit, so she found the wrong answer to Unit 3 Graded Assignment 4 Question 5. Bubba and Vince wrote the correct equation and found the correct answer for breaking even but they did not write the correct equation or find the correct answer for making a profit. Barbara, Günter, Jean just wrote the correct answer using words; they did not show their work. Mary did not answer this problem. Nan wrote the wrong the equations for both parts and consequently, did not find the correct answers.

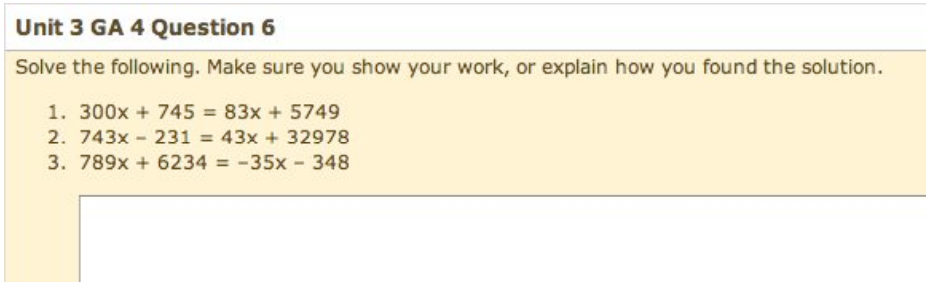


Figure 86: Unit 3 Graded Assignment 4 Question 6

Ana, Barbara, Günter, Jean, Mary, Nan, and Vince just wrote the answers to Unit 3 Graded Assignment 4 Question 6. Bubba showed his work but did not find the correct value for part 3; he added "-35x" to both sides (instead of "35x").

Interpreting Results of Equality

Table 16: Summery of Students' Understanding of Equality

	Interview Task	**11 Graded Assignment Problems**	**4 Post-test Questions**
Least Correct	1 responded "17"	All of the students copied the answers to U3GA1 Q4 None of the students correctly answered U3GA4 Q5 1 student correctly answered U3GA4 Q6	6 students correctly answered questions 29 and 31
	5 responded "12"	3 students correctly answered U3GA4 Q2 4 students correctly answered U3GA2 Q5 and U3GA4 Q3	7 students correctly answered question 36
Most Correct	2 responded correctly by writing "7"	5 students correctly answered U3GA4 Q1 6 students correctly answered U3GA2 Q4, U3GA2 Q6, and U3GA1 Q6 8 students partially answered U3GA4 Q4	All students correctly answered question 35

Here is my interpretation of the data. Günter and Jean were the only students to correctly solve the interview task about equality. They also correctly answered the 4 post-test questions. Of the 10 GA, Jean correctly responded to 5 of them and Günter correctly responded to 3 of them. At the other end of the spectrum, Mary answered "17" to the interview question and correctly answered only 2 of the 4 post-test items. Mary also correctly responded to 2 GA out of the 10 problems. The other five students scored somewhere in between. Bubba was able to answer all of the post-test items and 6 of the 10 GA correctly. Ana correctly answered all of the post-test items, and 5 of the 10 GA correctly. Barbara correctly answered all of the post-test items and 4 of the 10 GA

correctly. Nan correctly answered 1 of the post-test items and 3 of the 10 GA, Vince correctly answered 2 of the post-test items and 3 of the 10 GA.

Overall, the students' scores on the post-test improved but they did not answer many of the GA correctly. Even Bubba's score of 6 correctly answered out of 10 GA is not stellar. The students are probably better at solving equations than they were before taking the ASK ME-Algebra course but the data does not seem to indicate that the students are proficient in their understanding of equality.

Günter and Jean demonstrated that they have a relational understanding of equality from their responses to the interview task. The other six students demonstrated they have an operational understanding of equality (Falkner, et al., 1999).

Equality was assessed during the time that the teacher was helping the students. It is possible that the teacher allowed the students to not save the activities, which is where the bulk of the curriculum is located.

There is one set of data left to look at: the survey data. The next section discusses the results of the pre- and post-survey the treatment group students finished.

Views About Mathematics Survey (VAMS) Results

Seven students completed both the pre-survey and the post-survey. The VAMS measures six constructs that change over time. The constructs correspond to my second research question, "In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical

thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics?" (see Table 17)[37].

[37] Questions 7, 12, 20, and 29 – 35 did not have any expert, mixed or naïve delineations so they are not included in Table 5.

Table 17: Second Research Questions Constructs

	Interview	Survey Questions
RQ2. Learnability of mathematics	In which mathematics class did you get your best math grade? Worst math grade?	5, 9, 11, 13, 19, 34*
RQ2. Personal relevance	What kinds of grades do you generally earn in math class?	6, 7*, 20*, 27, 33*, 36
RQ2. Reflective Thinking	What do you think makes someone good at algebra?	10, 17, 18, 24, 42
RQ2. Structure of Knowledge	Which math concepts are you good at understanding?	14, 25, 30*, 32*, 35*, 39, 40
RQ2. Methods of mathematics		12, 16, 23, 29, 37, 38, 41
RQ2. Validity of Knowledge	Do you use algebra in any other classes? Which ones?	8, 15, 22, 26, 28, 31*

Six questions from VAMS included the responses from the students about their views of the "learnability" of mathematics before their experience with ASK ME-Algebra and after working on one semester of material. Appendix H includes the students' responses to the pre- and post-survey questions about their views of the "learnability" of mathematics.

Six questions from VAMS included the responses of the students to questions that indicate their views about the personal relevance of mathematics. Appendix I includes the students' responses to the pre- and post-survey questions about their views about the personal relevance of mathematics.

Five questions from VAMS included all of the responses students made to questions that deal with their views of the role of reflective thinking in mathematics.

Appendix J includes the students' responses to the pre- and post-survey questions about their views of the role of reflective thinking in mathematics.

Seven questions included students' responses that deal with the students' views of the structure of knowledge in mathematics and seven questions included the students' responses to questions students' views of the methods of mathematics. Appendix K includes the students' responses to the pre- and post-survey questions of the structure and methods of mathematics.

Six questions from VAMS involved the students' views about the validity of mathematics. Appendix L includes the students' responses to the pre- and post-survey questions of the students' views about the validity of mathematics.

The survey consisted of self-reported data from seven students. Therefore, the conclusions from this analysis cannot be generalized to the population of Algebra students in the state of Texas. The analysis of the aggregated data showed that one student was classified as "lower transitional" and the other six were "Naïve" before the treatment. Nine months later, all of the students were classified as "Naïve". A closer look at the data was needed. Refer to Appendices H through L. The analysis will be discussed in the follow sections.

For a student's views about mathematics to improve then their Folk views need to decrease and their Mixed and/or Expert views need to increase. When a student's views worsen then their Folk views increase and their Expert views decrease.

In Appendix H, the data for the questions about their views of the "learnability" of mathematics was stored. After treatment, students had very few Expert views but

more Mixed views. This can also be seen in the initial interview question that asked about their worst grade in a mathematics class, many students stated, “Algebra”.

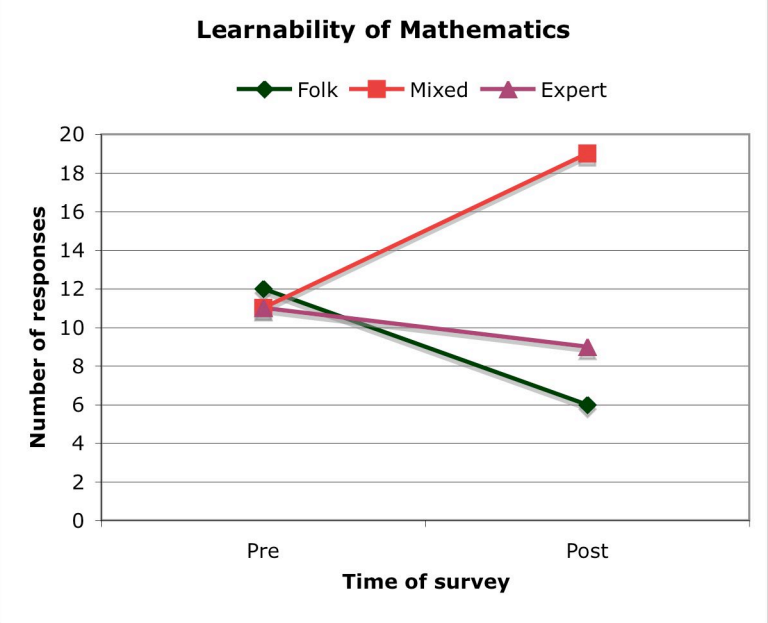


Figure 87: All students’ views of the “Learnability” of Mathematics

Overall, the students did not improve their views of the “learnability” of mathematics nor did their views worsen – as evidenced by the decrease in Folk views and the decrease in Expert views. This can be attributed to the mixed signals the students received while interacting with the curriculum for three months then working with the teacher on the same curriculum for six months.

Appendix I contains the data for the questions dealing with students views about the personal relevance of mathematics.

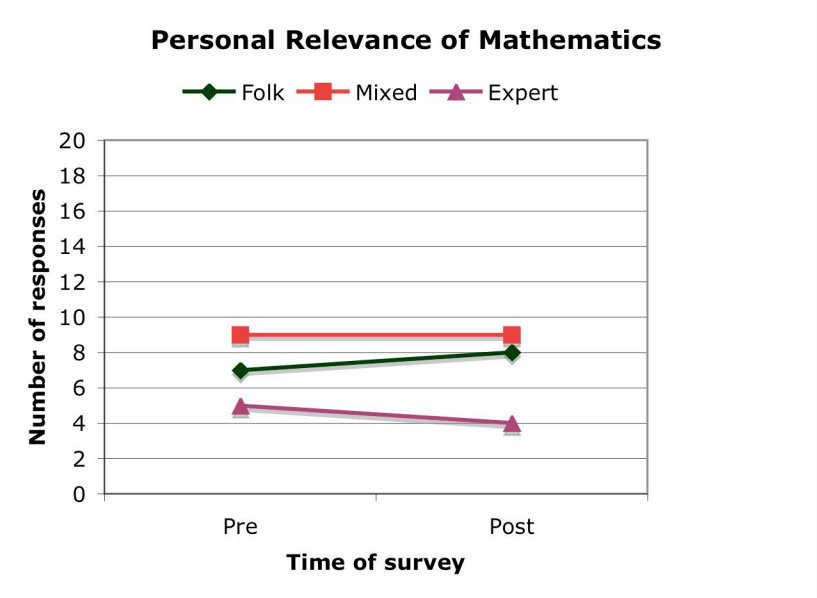


Figure 88: All students' views of the Personal Relevance of Mathematics

Overall, there seems to be a slight shift in the students' views to the view that mathematics is not personally relevant to them – as evidenced by the slight increase in Folk views and the slight decrease in Expert views. Many of the problems the students worked with throughout the curriculum were realistic. Very few of the students had experienced many of the situations that were presented in the problems in the course.

In Appendix J, the students' views of the role of reflective thinking in mathematics are documented.

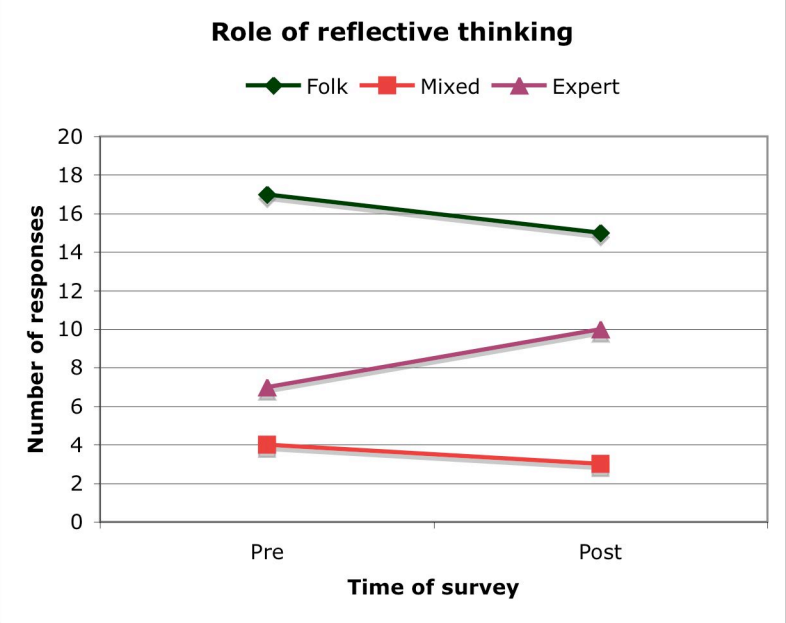


Figure 89: All students' views of the Role of Reflective Thinking

Overall, the students' views increased about the role of reflective thinking in mathematics. This can be attributed to the types of questions the students had to answer throughout the course. I postulate that the teacher even modeled this type of behavior when working the problems with the students.

Appendix K contains the results to questions of the structure and methods of mathematics.

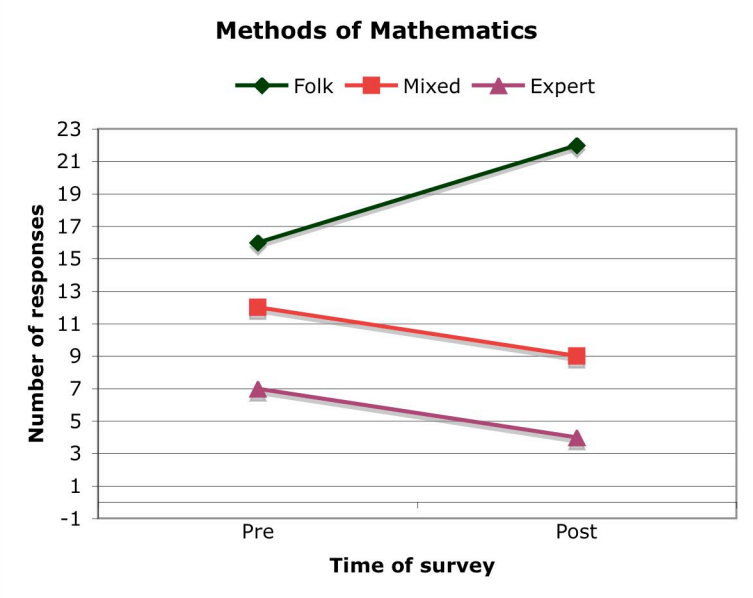


Figure 90: All students' views about the Methods of Mathematics

Overall, the students' views about the methods of mathematics decreased dramatically as seen by the 3 point drop in Expert views and the 6 point rise in Folk views. This can be attributed to the dichotomous methods of teaching mathematics the students encountered during the course of treatment.

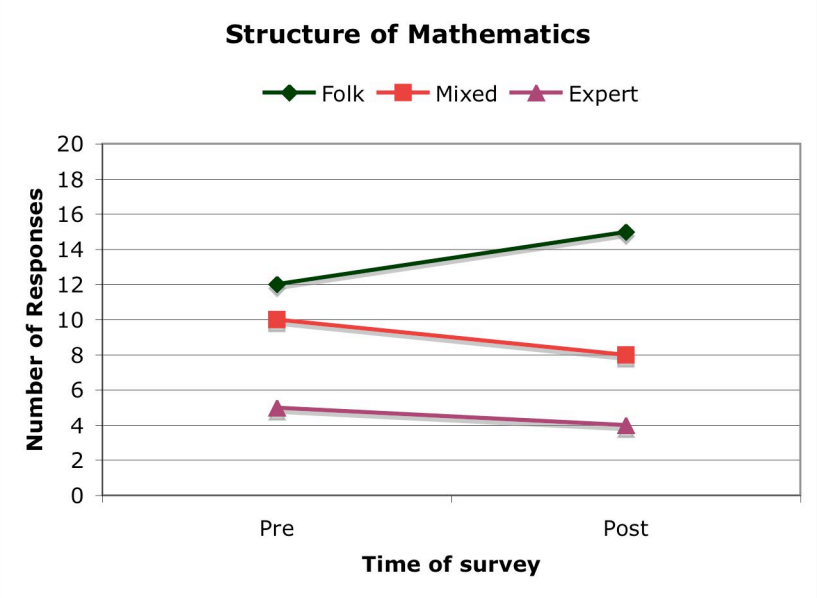


Figure 91: All students' views of the Structure of Mathematics

Overall, the students' views of the Structure of Mathematics worsened as evidenced by the decrease in their Expert views and the increase in their Folk views. This can be attributed to the students experience with course and the different ways the material was presented to them by the computer and then by the teacher.

The six questions that assessed students' views of the Validity of Mathematics were 8, 15, 22, 26, 28, and 31. The students' results to these questions are in Appendix L.

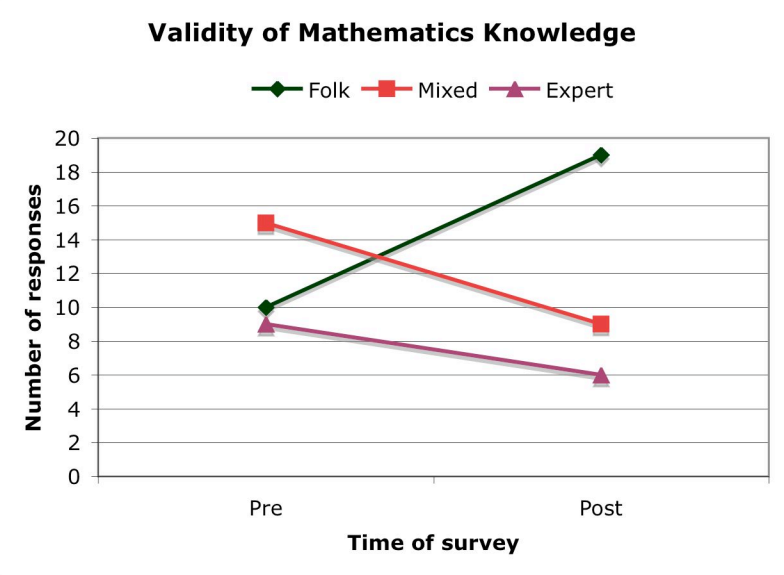


Figure 92: All students' views about the Validity of Mathematics Knowledge

Overall, the students' view of the validity of mathematics knowledge worsened – as evidenced by the large increase in Folk views and the decrease in Expert views. This can be attributed to the inaccuracies in grading the students' responses to the Graded Assignments. Many of the students stopped thinking during the last Unit of the course as evidenced by the increase in copied answers – even incorrectly answered responses.

Since students' views normally do not change in a short period, and this group of seven students did change their views negatively (i.e. from Expert or Mixed to Folk), their experience using the curriculum affected them strongly. Since the teacher intervened after three months, the students did not use the curriculum fully for six months. Therefore, the negative change in their views about mathematics cannot be attributed to the curriculum.

Most of the students struggled with the curriculum as evidenced by their lack of progress by October 2004 (only two of the students were working in Unit 2). The students were interacting with the curriculum as evidenced by the work that was saved by mid-October. The students were "on their own" and using the curriculum from mid-August until late October. There were 39 activities in the three units of the first semester of ASK ME-Algebra and none of the students saved more than 16 activities (about 41%). The pattern of not saving work seems to coincide with the teacher beginning to help the students twice a week in late October (see Table 18).

Table 18: Numbers of saved work before and after 10/24/04

Student	Number of Activities saved **Before** 10/24/04	Number of Activities saved **After** 10/24/04
Ana	None	6 (enrolled in December 2004)
Bubba	9	5
Barbara	12	2
Günter	10	None
Jean	6	5
Mary	6	None
Nan	16	None
Vince	6	7

Chapter 5 What does this all mean?

This study was designed to determine if the ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) instruction will help students learn three basic algebraic concepts: variables, expressions, and equality. This study looked at 8 students' work while using the ASK ME-Algebra curriculum and compared their post-test scores to the test scores of students in freshman algebra classes.

First, here is a reminder of the research questions for this study:

- In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' understanding of concepts, such as variables, equality, and equivalent expressions?
- In what ways could ASK ME-Algebra (or an online comprehensive and all-encompassing curriculum for Algebra I) influence students' views about the personal relevance of mathematics, the "learnability" of mathematics, the use of critical thinking in mathematics, the validity of mathematical knowledge, the structure of mathematical knowledge, and the methodology of mathematics?

The quantitative results are striking. There is a significant difference between the pre- and post-tests for the treatment group and there is a significant difference between the control group's test scores and the treatment group's post-test scores. Are the quantitative results too good to be true? It seems that way.

The qualitative results ring a cautionary tone. From the follow-up interview, the students could not determine if $x - y = z - y$ is always true, sometimes true or never true

and they could not simplify 3a – (b + a). Only two of the eight students could correctly find the value in the box of 8 + 4 = $\square$ + 5.

Triangulating the data

Did the treatment group learn about variables, expressions, and equality by working through the ASK ME-Algebra course? The data seem to be conflicting. The focus of this section will be to look at the data (interview tasks, the TAKS test items from the post-test and questions from the Graded Assignments that specifically address variables, expressions or equality) and find any correlation. First, a look at the students' understanding of variables will take place. Next, there will be a look at the students' understanding of expressions. Finally, there will be a look at the students' understanding of equality.

Discussion of Conclusions

The students' understanding of expressions and equality did not seem to improve due to having taken the ASK ME-Algebra course. The students did seem to improve their understanding of variables after working through the curriculum.

Were outside influences involved in students' non-success? Yes, the teacher did more instruction, the students saved less work, and the students did more copying of each other's answers after late October 2004. The students did not use the curriculum fully and relied on the teacher for assistance in answering the GA after October. The students either worked together in groups of three or four, or one student worked and the other students copied his or her work.

The teacher admitted that the students started copying everything written on the board or said by her during those 30-minute time-periods of instruction. Very little evidence of individual work was entered into the text boxes for answers in the GA of Unit 3.

The students saved very little work in Unit 3, which is reflected in the incomplete responses of the GA in Unit 3. Unit 3 covered the concept of equality and expressions. This could explain the lack of student improvement in their understanding of equality and expressions.

The teacher stated on 10/19/04 that she would be helping the students for 30-minutes at a time twice a week after the TAKS administration the next week. After looking at the dates of saved work, most of the students had more activities saved before 10/24/04 than after that date (see Table 6).

The teacher had a big influence on the students' performance. The students stopped saving their responses to the Guided Practice and possibly stopped viewing and interacting with the tutorials. The students stopped showing work on many of the problems. They copied down anything that they read or heard, even the typos or incorrect responses.

A factor that coincides with how the quantitative data was so significantly different is the fact that the treatment group answered the "post-test" questions a few at a time as they covered the material in the courses. The control group answered all 19-questions at one time.

The treatment group also possibly had the opportunity to redo the multiple-choice "post-test" questions, which can explain the significant differences between the pre- and post-test scores of the treatment group. The treatment group did answer the pre-test questions all at one time.

The students did improve their knowledge of variables but not expressions and equality. Is it possible that the students had a very long "learning curve" for the new type of instruction? Could the students have succeeded with the concepts of expressions and equality if the teacher had not intervened? Possibly, but we will never know for sure for these eight students.

Since the data seem to indicate that the students did not use the full curriculum, the students' worsening of their views towards mathematics is understandable. The students' views of the "learnability" of mathematics stayed the same; the students' views about the personal relevance of mathematics slightly worsened; students' views of the role of reflective thinking increased during treatment; students' views about the methods of mathematics worsened; students' views about the structure of mathematics worsened during treatment; students' views about the validity of mathematics knowledge also worsened during treatment.

There is a dramatic change in students' views about mathematics after 7 to 9 months of treatment, which has not occurred so quickly in past research. The treatment of using the ASK ME-Algebra curriculum is not likely the cause. The students only used the curriculum as independent learners for 3 of 9 months.

Comments received from two of the students in their first and second interviews were very positive. The students like to read, to type their answers onto the computer (instead of writing on paper), to access the glossary with one click, and to review answers stored on the computer. Both students finished in March. Their upbeat view of the course assisted them to complete the material.

Comments from two other students during the first and second interviews were negative. They did not like to read, to type, to work word problems, and to think hard. A comment that they both made was "why can't we do problems like in the book?" One of these students did not complete the course.

Research has shown that students views do not change over one semester of curriculum using "an early introduction to functions with an emphasis on applied problems" (Carlson, 1997a, p. 27). Not only did the eight students' views about mathematics change after working with ASK ME-Algebra curriculum but most of their views worsened (i.e. changed from Mixed or Expert to Folk).

The teacher's intervention was shown to have had negative affects on the students' use of the curriculum, and it is possible that the teacher's intervention also negatively influenced the students' views towards mathematics.

Can the ASK ME-Algebra curriculum help all students learn algebra? This study's results cannot be generalized to answer that question but future studies of this curriculum can attempt to answer that question.

Chapter 6 Limitations and Future Directions

This chapter includes a discussion of the limitations of the current study. A discussion of future directions to take in studying the ASK ME-Algebra curriculum also appears. Finally, I offer some thoughts on what teachers should know before using this curriculum in order to instruct students so that students can be successful in understanding the concepts of variables, expressions, and equality.

Limitations

Since this was a case study of eight students, the results cannot be generalized to the population of high school algebra students, but the experiences of these eight students can enlighten others. The students in this study were older than most students enrolled in Texas high school Algebra I courses. While the students had seen many of the topics in previous classes, they had not been successful in mastering variables, expressions and equality.

Students in classrooms with teachers will have different experiences than students in a lab setting or by themselves at home. The students in this study did not have the teacher's full attention every day of the week. The teacher admitted to working with the students twice per week for 30 minutes at a time after the October administration of the TAKS test.

Students who have experienced having algebra taught algorithmically will be at a disadvantage when working with this curriculum. Student may expect instruction similar to their past experiences but the ASK ME-Algebra curriculum is not similar to a lecture-based or procedural-based instruction. Many of the students in this study struggled with adjusting to the different approach of the curriculum. The students worked independently until the teacher assisted the students in a manner to which they were accustomed.

The students in this study tended to not perform work that was not "graded" by the teacher. When the students and teacher worked together, the students copied word for word what was written on the board or said by the teacher.

The quantitative data of the post-test from the treatment group was not optimally collected which was beyond my control. The treatment group had the option to redo the questions multiple times but the control group did not. If this study is replicated then the two groups should take the test in the same format.

Future Directions

Replication of this study is warranted. Currently, there are 46 students using the curriculum as a correspondence course in order to receive credit for high school algebra. Most of the students are using the curriculum at home alone. The students are from all over the United States.

Teachers are using the curriculum in their classrooms across Texas in many different ways. One concern is: what is the most effective method of using the curriculum in the classroom in order for students to gain a better understanding the concepts of variables, expressions, and equality?

One area warranting examination that was not included in this study would be to observe student interaction with the ASK ME-Algebra curriculum on a regular basis. This would be done in order to verify that students utilize all components of the curriculum to its fullest.

Another area in which a future study would permit improvement over this study would be to devise contextual interview tasks. The ASK ME-Algebra curriculum is contextual and in order to determine if the students understand the concepts of variables, expressions, and equality then the interview tasks should be contextual.

Any future study should include a collection of TAKS released items or similar items. These items can be randomly assigned to students to answer either during the course or at the end of the course as a post-test. This process could assure that the student answers the post-test questions by themselves.

A professional development workshop on ASK ME-Algebra

Before using the ASK ME-Algebra curriculum teachers should understand the functions-based approach to algebra. How does a functions-based approach differ from the textbook or other curriculum used in their classrooms? Once the differences between the curricula are understood, then the teachers should determine for themselves if they can teach algebra with the functions-based approach. If teachers can use the functions-based approach to teach algebra then the next obstacle to overcome would be the adaptation of Problem-Based Instruction (PBI).

Teachers should understand what PBI encompasses. The use of anchor videos to explain each problem in the unit is an important part of PBI. The "guided practice" in which students attempt to solve the problems, save their answers, check their work, and revise their work is another integral part of PBI. The formative assessment tool in ASK ME-Algebra is the Graded Assignments. Another important part of PBI is to allow students to revise their work, so that each question in the GA is capable of allowing a student to redo it (if the teacher requests it).

If time allows then the teachers should work through every aspect of the curriculum. The approach each teacher plans to use in the classroom (i.e. with the use of group work or individual work) should be implemented as the teachers work through the course. In other words, if a teacher plans to have their students work cooperatively then he or she should work cooperatively with other teachers through the course.

There are many ways in which teachers can incorporate the ASK ME-Algebra into their current teaching practices. The availability of technology in each school and the limitations of that school's technology would determine how the curriculum could be most effectively utilized. First, the teacher would need to determine if the school's computers are capable of handling the versions of Shockwave and Flash required by the curriculum. Next, the teachers would need to determine if the school's infrastructure can handle up to 40 computers downloading videos at the same time.

Relevance to the literature

This is the first study of a functions-based approach to algebra at the high school level that has included a long-term (18 weeks or more) study period and emphasized the understanding of three concepts: variables, expressions and equality.

Algebra is an important first step in the mathematics career of most high school students. A new method of algebra instruction has various effects on students' learning, understanding, attitudes and motivation. The students' understanding of variables can improve and the students' attitudes regarding mathematics can change when learning within a different environment. This study took a long-term look at students as they interact with a new curriculum and a new method of instruction.

Based on the results of this study, what can be done to better prepare students to be successful in the "stumbling block" course of high school? Using PBI can facilitate students to observe that algebra is useful outside the classroom, which in turn assists students in learning the material.

The intentions of this researcher were to incorporate the best methods from research to use in the classroom with students, and implement the goals of administrators and state legislature members into the schools.

Appendix A Consent Forms

Parent Consent Form for Children Enrolled in Online Algebra 1 Course through the Distance Education Center, page 1

CONSENT FORM

Evaluation of Online Algebra I Materials

Your child is being invited to participate in a study of how technology can help students learn math. This study is being conducted by Joey Offer of the Distance Education Center in Continuing and Extended Education and Professor Susan Williams of UT Austin's College of Education. The Distance Education Center has developed high school Algebra I materials. This study will investigate the problem solving strategies that students use when using these materials. Identifying these strategies will help the Distance Education Center improve their online Algebra I courses.

If you allow your child to participate, we will provide him or her with the necessary materials (note: the school district will supply a graphing calculator if needed). These materials will include complex real world problems, computer software, and problem solving exercises and assessments. All of the materials are compliant with the Texas Essential Knowledge and Skills and were developed by Texas certified teachers working with the Distance Education Center which is accredited by the Southern Association of Colleges and Schools. Your child will work on their own at their own pace through the materials, but he or she will have access to a mathematics teacher by telephone or by the Internet during set times.

We will monitor the work your child turns in to the Distance Education Center. We may also request an interview at the end of each lesson if we need to ask your child about any of the strategies we observed in the work. We may also ask your child to work some problems and talk about the strategies he or she used.

The interviews will be videotaped or audio taped. Videotapes and audiotapes of the children will be stored in a locked office belonging to this project and viewed only by researchers working on the project. Analysis of these tapes is for research purposes only and will not affect your child's grades. The tapes may be kept for possible future analysis. They will be labeled so that no personally identifying information is visible on them.

Any information that is obtained in connection with this study and that can be identified with your child will remain confidential and will be disclosed only with your permission. His or her responses will not be linked to his or her name or your name in any written or verbal report of this research project.

There are no physical risks and injuries anticipated in this study; however, no treatment will be provided for research related injury and no payment can be provided in the event of a medical problem.

Authorized persons from The University of Texas at Austin and the Institutional Review Board have the legal right to review research records and will protect the confidentiality of those records to the extent permitted by law. If the research project is sponsored then the sponsor also has the legal right to review your research records. Otherwise, your child's research records will not be released without your consent unless required by law

or a court order. No guarantees can be made, however, regarding the interception of data sent via the Internet by any third parties.

Your decision to allow your child to participate will not affect your or his or her present or future relationship with The University of Texas at Austin. If you wish your child to stop participation in this research study for any reason or if you have any questions, you should contact: Joey Offer at (512) 471-8913.

If you have any questions or concerns about your child's participation in this study, call Professor Clarke Burnham, Chair of the University of Texas at Austin Institutional Review Board for the Protection of Human Research Participants at 232-4383.

You may keep the copy of this consent form.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. If you later decide that you wish to withdraw your permission for your child to participate in the study, simply tell me. You may discontinue his or her participation at any time.

_____
Printed Name of Child

_____ _____
Signature of Parent(s) or Legal Guardian Date

_____ _____

_____ _____

_____ _____
Signatures of Principle Investigators Date

If your child is between the ages of 13 and 17, then he/she may use this form to give his or her assent to participate in this study. If your child is under the age of 12 years, then a separate assent form will be provided.

I have read the description of the study titled Evaluation of Problem-Based Algebra I Materials that is printed above, and I understand what the procedures are and what will happen to me in the study. I have received permission from my parent(s) to participate in the study, and I agree to participate in it. I know that I can quit the study at any time.

_____ _____
Child's Signature Date

We may wish to present some of the tapes from this study at scientific conventions or as demonstrations in classrooms. Please sign below if you are willing to allow us to do so with the tape of your child's performance.

Parent Consent Form for Children Enrolled in Online Algebra 1 Course through the Distance Education Center, page 3

I hereby give permission for the videotape made for this research study to be also used for educational purposes.

_____ _____
Signature of Parent or Legal Guardian Date

Parent Consent Form for Children Enrolled in Algebra 1 Course through
the Lockhart Freshman Campus, page 1
CONSENT FORM
Evaluation of Algebra I Materials

Your child is being invited to participate in a study of how textbooks can help students learn math. This study, conducted by Jane Ries Cushman and Jenn Smith of the University of Texas at Austin's College of Education, will investigate the problem solving strategies that students use when using the textbook. To participate your child will take an assessment of 19 multiple-choice questions. Identifying these strategies will help Jane Ries Cushman complete her dissertation.

If you allow your child to participate, we will provide him or her with the necessary materials (note: the school will supply a graphing calculator, if needed). These materials will include complex real world problems and problem solving exercises. All of the materials are compliant with the Texas Essential Knowledge and Skills and were developed by Texas certified teachers working with the Distance Education Center of the University of Texas, which is accredited by the Southern Association of Colleges and Schools. Your child will work on their own at their own pace through the materials.

We may also request an interview at the end of the assessment if we need to ask your child about any of the strategies we observed in their work. We may also ask your child to work more problems and talk about the strategies he or she used.

The interviews will be video taped or audio taped. Videotapes and audiotapes of the children will be stored in a locked office belonging to this project and viewed only by researchers working on the project. Analysis of these tapes is for research purposes only and will not affect your child's grades. The tapes may be kept for possible future analysis. They will be labeled so that no personally identifying information is visible on them.

Any information that is obtained in connection with this study and that can be identified with your child will remain confidential and will be disclosed only with your permission. His or her responses will not be linked to his or her name or your name in any written or verbal report of this research project.

There are no physical risks and injuries anticipated in this study; however, no treatment will be provided for research related injury and no payment can be provided in the event of a medical problem.

Authorized persons from The University of Texas at Austin and the Institutional Review Board have the legal right to review research records and will protect the confidentiality of those records to the extent permitted by law. If the research project is sponsored then the sponsor also has the legal right to review your research records. Otherwise, your child's research records will not be released without your consent unless required by law or a court order. No guarantees can be made, however, regarding the interception of data sent via the Internet by any third parties.

Your decision to allow your child to participate will not affect your or his or her present or future relationship with The University of Texas at Austin. If you wish your child to stop participation in this research study for any reason or if you have any questions, you should contact: Jane Ries at (512) 232-2877.

Parent Consent Form for Children Enrolled in Algebra 1 Course through the Lockhart Freshman Campus, page 2

If you have any questions or concerns about your child's participation in this study, call Professor Clarke Burnham, Chair of the University of Texas at Austin Institutional Review Board for the Protection of Human Research Participants at (512) 232-4383.

You may keep a copy of this consent form. A copy will be made once all of the signatures are collected.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. If you later decide that you wish to withdraw your permission for your child to participate in the study, simply tell anyone involved in with the project or your child's teacher. You may discontinue his or her participation at any time.

_____
Printed Name of Child

_____ _____
Signature of Parent(s) or Legal Guardian Date

_____ _____

_____ _____
Signatures of Principle Investigators Date

If your child is between the ages of 13 and 17, then he/she may use the form below to give his or her assent to participate in this study. If your child is under the age of 12 years, then a separate assent form will be provided.

I have read the description of the study titled Evaluation of Algebra I Materials that is printed above, and I understand what the procedures are and what will happen to me in the study. I have received permission from my parent(s) to participate in the study, and I agree to participate in it. I know that I can quit the study at any time.

_____ _____
Child's Signature Date

We may wish to present some of the tapes from this study at scientific conventions or as demonstrations in classrooms. Please sign below if you are willing to allow us to do so with the tape of your child's performance.

I hereby give permission for the audio/videotape made for this research study to be also used for educational purposes.

_____ _____

Signature of Parent or Legal Guardian Date

Appendix B Views About Mathematics Survey

Student id #_____

Views About Mathematics Survey

Form M 13

This survey is designed by the Modeling Instruction and ACEPT research teams at Arizona State University. It is intended to identify factors that affect people's understanding of mathematics, and to assist in the design of instructional material.

*Your participation is **voluntary**. The results will not affect your grade, even if you choose not to participate. All data are **confidential**. Your identity will not be disclosed to any party.*

Please:

*Use a **No. 2 pencil**.*

*Make **only one** mark per item.*

*Do **not** skip any question.*

Avoid guessing. Your answers should reflect what you actually and honestly think.

Plan to finish the survey in 30 minutes.

I. *Please use the following rating scale to answer the next four questions, and mark your answers in the space at the beginning of each statement.*

A: Excellent; **B:** Good; **C:** Average; **D:** Weak; **E:** Extremely Poor; **F:** Never studied

In the class in which you are taking the survey, rate how well you **expect** to do in the following:

_____ 1. Understanding the material presented

_____ 2. Solving homework problems on your own

_____ 3. Exams

_____ 4. How do you currently rate your overall confidence in your mathematical abilities?

II. What is your age? _____

A: 13 – 15 **B:** 16 – 18 **C:** 19 – 21 **D:** 21 – 31 **E:** 32 or over

III. *The example below illustrates the seven choices that you have for answering questions 1 through 31 in this section. Please circle your choice to these questions.*

Example

Learning mathematics requires (a) a serious effort (b) a special talent.

Only (a)	Mostly (a)	equally (a) & (b)	Mostly (b)	Only (b)
1	2	3 4 5	6	7

What would each one of the seven choices mean?

Only (a), Never (b): Learning mathematics requires ***only*** *a serious effort* and **no** special talent *at all*.

Mostly (a), Rarely (b): Learning mathematics requires ***far more*** *a serious effort* than a special talent.

More (a) Than (b): Learning mathematics requires ***somewhat more*** *a serious effort* than a special talent.

Equally (a) & (b): Learning mathematics ***equally*** requires ***both*** a serious effort and a special talent.

More (b) Than (a): Learning mathematics requires ***somewhat more*** a *special talent* than a serious effort.

Mostly (b), Rarely (a): Learning mathematics requires ***far more*** a *special talent* than a serious effort.

Only (b), Never (a): Learning mathematics requires ***only*** a *special talent* and no serious effort *at all*.

5. Learning mathematics requires (a) a serious effort (b) a special talent.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

6. If I had a choice: (a) I would never take another mathematics class (b) I would still take mathematics for my own benefit.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

7. Reasoning skills that are taught in mathematics classes can be helpful to me (a)in my everyday life (b) if I were to major in mathematics or a related field.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

8. My score on mathematics exams is a measure of how well (a) I understand the covered material (b) I can do things the way they are done by the teacher or in some class materials.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

9. For me, doing well in mathematics classes depends on (a) how much effort I put into studying (b) how well the teacher explains things in class.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

10. When I experience a difficulty while doing mathematics (a) I quickly seek help, or give up trying (b) I try hard to figure it out on my own.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

11. When studying mathematics in a textbook or in class materials (a) I memorize it the way it is presented (b) I make sense of the material so that I can understand it.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

12. Graphing calculators and computers (a) bring new methods for solving mathematics problems (b) speed up problem solving using established methods.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

13. In mathematics, it is important for me to (a) memorize technical terms and mathematical formulas (b) understand the ideas and when and how to use them.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

14. The purpose of mathematical formulas is to (a) express meaningful relationships among variables (b) provide ways to get numerical answers to problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

15. After I go through a mathematics textbook or class materials and feel that I understand them (a) I can solve related problems on my own (b) I have difficulty solving related problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

16. The first thing I do when solving a word problem that involves mathematics is (a) represent the situation with sketches and drawings (b) search for formulas that relate givens to unknowns.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

17. In order to solve a mathematics problem (a) I need to have seen the solution to a similar problem before (b) I apply general problem solving techniques.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

18. Seeing alternate solutions to a mathematics problem is (a) a waste of my time (b) helpful for improving my problem solving abilities.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

19. How well I do on mathematics exams depends on how well I can (a) recall material in the way it was presented in class (b) do tasks that are somewhat different from ones I have seen before.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

20. Using graphing calculators or computers (a) increases my interest in studying mathematics (b) is a waste of my time.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

21. Mathematical functions that represent relationships in the physical world are (a) exact expressions of what is being represented (b) approximate expressions of what is being represented.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

22. The relationship among the sides of a right triangle expressed in the Pythagorean theorem is true because it has been (a) proven by a logical argument (b) verified by measurement.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

23. Collecting and graphing real world data is useful for (a)determining patterns and making general predictions (b) obtaining numerical answers to specific problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

24. For me, making unsuccessful attempts when solving a mathematics problem is (a) a natural part of my pursuit of a solution to the problem (b) an indication of my weaknesses in mathematics.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

25. When solving a challenging mathematics problem, a mathematician (a) makes many incorrect attempts (b) moves directly to a correct solution.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

26. The process of attempting to solve a problem that involves mathematical reasoning is (a) a satisfying experience (b) not a satisfying experience.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

27. For me, the relationship of mathematics classes to everyday life is usually (a)easy to recognize (b) hard to recognize.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

28. Proving a mathematical theorem requires (a) evidence from the physical world (b) a logically sound argument.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

29. The role of a mathematics teacher is to (a) show me how to work specific problems (b) guide me in learning to solve problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

30. The focus of student learning should be on (a) memorizing specific content (b) assimilating information from class and the textbook.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

31. A student's ability in mathematics is demonstrated by (a) making logically sound arguments (b) producing content knowledge.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

32. A major goal of mathematics instruction is to (a) impart information (b) equip students to solve problems independently.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

33. When completing a lesson in mathematics, I need to (a) use only mathematical symbols (b) write mathematics using words and mathematical symbols.

Only (a)	Mostly (a)	equally (a) & (b)	Mostly (b)	Only (b)

1 2 3 4 5 6 7

34. In solving mathematics problems, graphing calculators or computers help me (a) understand the underlying mathematical ideas (b) obtain numerical answers to problems.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

35. After a theorem has been proven and accepted in mathematics (a) it will never be changed (b) it may be rejected at a future time.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

36. I study mathematics: (a) to satisfy graduation requirements (b) to learn useful knowledge.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

37. After I have answered all questions in a homework mathematics problem: (a) I stop working on the problem (b) I check my answers and the way I obtained them.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

38. After the teacher solves a mathematics problem for which I got a wrong solution: (a) I discard my solution and learn the one presented by the teacher (b) I try to figure out how the teacher's solution differs from mine.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

39. Different branches of mathematics, like geometry and algebra: (a) are related by common principles (b) have no relationship to one another.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

40. If we want to apply a method used for solving one mathematics problem to another problem, the objects involved in the two problems must be: (a) identical in all respects (b) similar in some respects.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

41. Scientists use mathematics as: (a) a tool for analyzing and communicating their ideas (b) a source of factual knowledge about the natural world.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

1 2 3 4 5 6 7

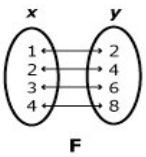
42. I answered all the questions in the survey: (a) to the best of my ability (b) without thinking seriously about them.

Only (a) Mostly (a) equally (a) & (b) Mostly (b) Only (b)

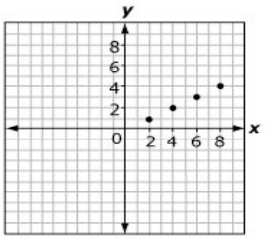
1 2 3 4 5 6 7

Appendix C TAKS released Algebra items; Pre-Test and Post-Test placement in the course

The function $f(x)=\{(1,2), (2,4), (3,6), (4,8)\}$ can be represented in several other ways. Which is NOT a correct representation of the function $f(x)$?



F



G

H x is a natural number less than 5 and y is twice x.

J $y = 2x$ and the domain is {1,2,3,4}.

- (•) F
- ( ) G
- ( ) H
- ( ) J

Figure 93: Question 2 on the pre-test and placed in the course at U1GA1Q8

The equation $c = 0.75t$ represents c the total cost of t tickets on a bus. Which table contains values that fit this equation?

F

Cost of Bus Tickets				
t	1	2	3	4
c	$.75	$1.50	$2.25	$3.00

G

Cost of Bus Tickets				
t	1	2	3	4
c	$.75	$1.00	$1.25	$1.50

H

Cost of Bus Tickets				
t	1	2	3	4
c	$1.75	$2.50	$3.25	$4.00

J

Cost of Bus Tickets				
t	1	2	3	4
c	$1.75	$2.75	$3.75	$4.75

Figure 94: Question 4 on the pre-test and placed in the course at U1GA2Q5

Students in a science class recorded lengths of a stretched spring, as shown in the table below.

Length of Stretched Spring

Distance Stretched, x (centimeters)	Weight, y (newtons)
0	0
2	10
4	20
7	35
9	45
10	50

Which equation best represents the relationship between distance stretched x and the weight on the spring y?

- F $y = -5x$
- G $y = 5/x$
- H $y = 5x^2$
- J $y = 5x$

Figure 95: Question 8 on the pre-test and placed in the course at U1GA3Q10

Which equation best describes the relationship between the corresponding values of x and y shown in the table?

x	y
−2	−12
0	−6
1	−3
4	6

- A $y=x-10$
- B $y=2x-8$
- C $y=3x-6$
- D $y=x^2-8$

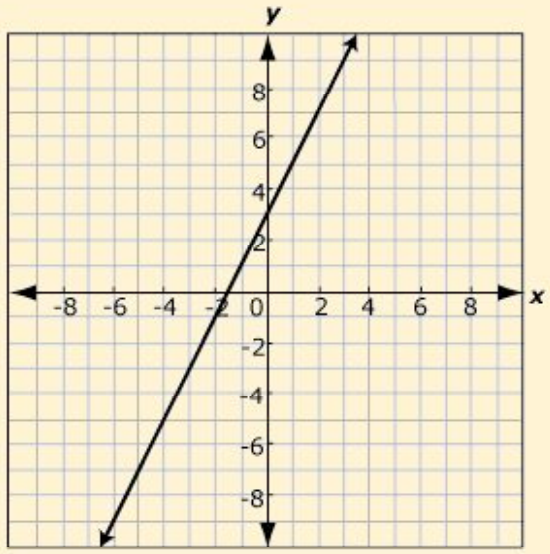
Figure 96: Question 9 on the pre-test and placed in the course at U1GA3Q9

Passengers on many commercial flights may make calls from a telephone provided by the airline. On a certain airline a call costs \$3 to connect plus \$2 for each minute. Which equation best represents c, the total cost for a call that lasts m minutes?

- F $m = 3 + 2c$
- G $c = 3 + 2m$
- H $m = 2 + 3c$
- J $c = 2 + 3m$

Figure 97: Question 10 on the pre-test and placed in the course at U1GA4Q3

The graph of the line $y = 2x + 3$ is drawn on the coordinate system below.



Which table of ordered pairs contains only points on this line?

F

x	y
-2	1
0	3
1	5
3	9

H

x	y
-1	-2
3	0
5	1
7	2

G

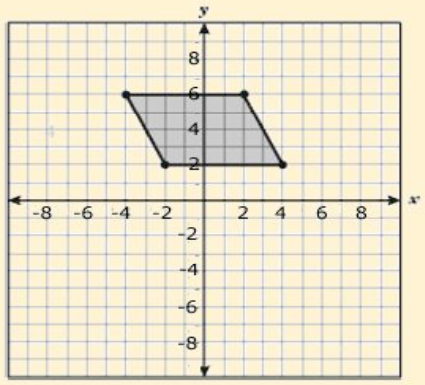
x	y
-2	-1
0	3
1	5
2	7

J

x	y
2	1
0	3
1	5
2	7

Figure 98: Question 12 on the pre-test and placed in the course at U1GA4Q5

A shaded parallelogram is graphed on the coordinate grid below.



Which of the following functions describes a line that would include an edge of the shaded parallelogram?

- F $y = -2x + 5$
- G $y = -2x - 2$
- H $y = -2x + 9$
- J $y = -2x - 1$

Figure 99: Question 20 on the pre-test and placed in the course at U2GA4Q7

Which function includes the data set {(2,4), (6,6), (12,9)}?

- F $y = 2x$
- G $y = \frac{x}{2}$
- H $y = 2x - 9$
- J $y = \frac{x}{2} + 3$

Figure 100: Question 22 on the pre-test and placed in the course at U2GA4Q9

A weather balloon is launched from a height of 475 feet above sea level. If the balloon rises at a constant rate of 85 feet per minute, which equation could be used to determine t, the time in minutes it will take the balloon to reach a height of 9,245 feet above sea level.

- A $9,245 = 85 + 475t$
- B $9,245 = 85(t + 475)$
- C $9,245 = 475 + 85t$
- D $9,245 = (475 + 85)t$

Figure 101: Question 23 on the pre-test and placed in the course at U2GA4Q10

A candy company sells chocolate-covered cherries in a box. The empty box weighs 4.2 ounces. Each piece of candy weighs at least 1.8 ounces. Which inequality best describes the total weight in ounces, w, of a box of chocolate-covered cherries in terms of c, the number of candies in the box?

- **F** $w \geq 1.8c + 4.2$
- **G** $w \geq 1.8c - 4.2$
- **H** $w \geq 4.2c + 1.8$
- **J** $w \geq 4.2c - 1.8$

Figure 102: Question 24 on the pre-test and placed in the course at U3GA1Q7

The area of a rectangle is $3x^2 + 14x + 8$, and the width is $x + 4$. Which expression best describes the rectangle's length?

- **A** $3x + 2$
- **B** $2x + 4$
- **C** $2x + 2$
- **D** $3x - 2$

Figure 103: Question 25 on the pre-test and placed in the course at U3GA2Q7

Simplify the algebraic expression $5(x + 3)(x + 2) - 3(x^2 + 2x + 1)$.

- **F** $2x^2 + 7$
- **G** $2x^2 + 27$
- **H** $2x^2 + 7x + 7$
- **J** $2x^2 + 19x + 27$

Figure 104: Question 26 on the pre-test and placed in the course at U3GA2Q8

Simplify the algebraic expression $3(x + 3) - 2(x + 3)$.

- **A** $x + 3$
- **B** $x - 3$
- **C** $-6x^2 - 54$
- **D** $6x^2 + 3$

Figure 105: Question 27 on the pre-test and placed in the course at U3GA2Q9

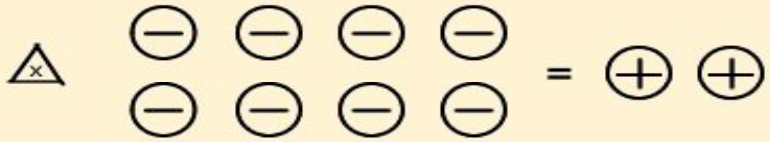
Which problem situation matches the equation below?

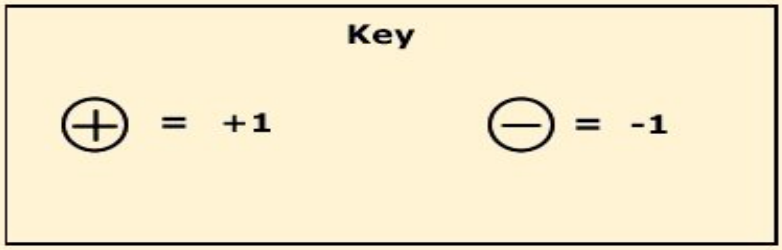
$x - 4.72 = 5.28$

- **A** Sergio's lunch cost $4.72. He receives $5.28 in change when he paid the bill. What is x, the amount of money he gave the cashier?
- **B** Yvette cycled 4.72 kilometers in a race. The winning cyclist's time was 5.28 seconds faster then Yvette's. What is x, the time in seconds it took Yvette to finish the race?
- **C** Janice and Maura measured the wingspans of butterflies in science class. Janice's butterfly had a wingspan of 4.72 centimeters, and Maura's butterfly had a wingspan 5.28 centimeters. What is x the average length of a butterfly's wingspan?
- **D** Mrs. Castro paid $4.72 for a jar of iced-tea mix that was originally priced at $5.28. What is x, the amont of money that Mrs. Castro saved altoghter?

Figure 106: Question 29 on the pre-test and placed in the course at U3GA3Q6

The model represents the equation





What is the value of x?

- **A** $x = -6$
- **B** $x = 4$
- **C** $x = 8$
- **D** $x = 10$

Figure 107: Question 31 on the pre-test and placed in the course at U3GA3Q8

The Childress family went on a camping trip. They paid $28.00 for a 2-night stay at a campground that allows a maximum stay of 30 nights. Which equation can they use to find c, the cost of camping at this campground for the maximum number of nights?

- **F** $c = 60 \cdot 56$
- **G** $c = 30 \cdot 28$
- **H** $c = 28 \cdot 28$
- **J** $c = 30 \cdot 14$

Figure 108: Question 32 on the pre-test and placed in the course at U3GA3Q9

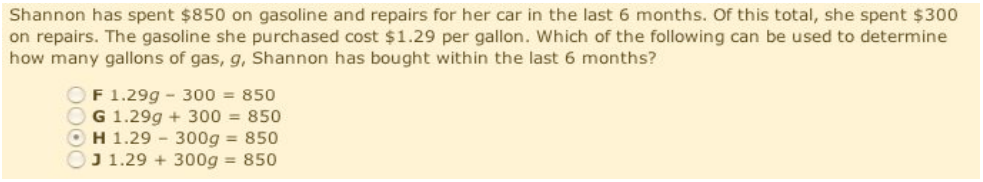


Figure 109: Question 34 on the pre-test and placed in the course at U3GA4Q8

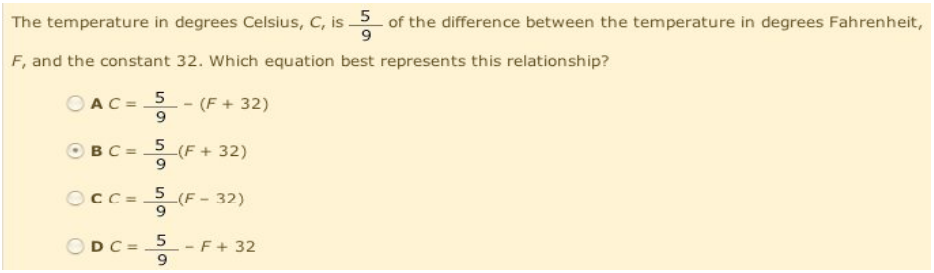


Figure 110: Question 35 on the pre-test and placed in the course at U3GA4Q9

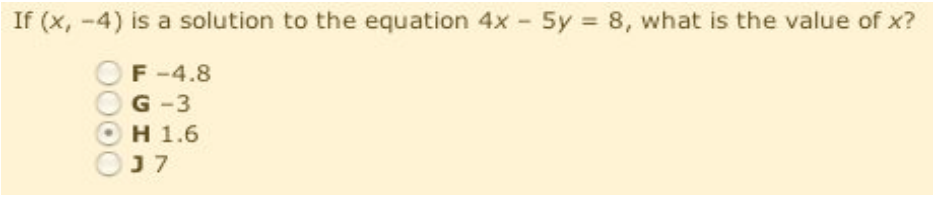


Figure 111: Question 36 on the pre-test and placed in the course at U3GA4Q10

Appendix D First Interview Protocol

Name _____ Pride High School

1. What are your hobbies, interests, or things you do in your spare time?

2. What was the highest level of mathematics you received credit for?
   6th grade math 7th grade math 8th grade math Algebra
   Geometry Math Models Algebra II

3. When did you complete (your last mathematics) course?

4. What schools have you attended?
   School Name (Public, Private, or Home?):

5. What kinds of grades do you generally earn in math class?
   A B C D F

6. How do you think you are doing in this class so far?

7. In which mathematics class did you get your best math grade? Worst math grade?
   Class Best Grade
   Class Worst Grade

8. Describe yourself as an algebra student.
   a. Do you like algebra?
   b. Why or why not?

9. What do you think makes someone good at algebra?
   a. Are you good at algebra?
   b. Why or why not?

10. Was the final project in Graded Assignment 5 a fair assessment of what you have learned about functions?

11. Which math concepts are you good at understanding?

12. Which math concepts are you not so good at understanding?

13. What are your plans after you graduate high school?

14. How far do you expect to go in taking mathematics courses in high school?

Geo & Math Models	Geo. & Alg. 2	Geo, Alg. 2 & pre – calculus	Geo, Alg. 2 calculus

15. Do you use algebra in any other classes? Which ones?

16. If your parents/guardians work outside the home, what kind of work do they do?

17. Do your parents/guardians like math?

18. Do you know what their highest levels of education are?

Elementary school High school College
Master's Degree Doctoral Degree (Ph.D., M.D., Ed.D., Etc.)

19. Have you sat through an Algebra 1 class before?

    a. If they respond yes: What do you notice is different about this course? (Write down each response to ask the following multiple times, if needed):

        i. Do you like or dislike _____? Why?

        ii. Do you like or dislike _____? Why?

        iii. Do you like or dislike _____? Why?

    b. If they respond no: What do you like about this course so far? What do you dislike so far?

20. Please tell me your definition of a function.

21. What do you think of the course so far?

22. Did you like the introduction video?

23. Did you like any of the tutorials?

(Function Relationships; Function Machine; Increasing, Decreasing and Constant Functions; Function Mapping; Graphing on Graph Paper; Graphing Calculator; Discrete and Continuous Functions; Two Second Rule; Process Column; Four Corner Model)

24. Did a tutorial help you understand that concept?

25. What other comments do you have about algebra and its importance in your life?

26. Do you have any questions for me?

Appendix E Second Interview Protocol

Name _____ Pride High School

1. How do you think you are doing in this class so far?

2. Describe yourself as an algebra student.
   a. Do you like algebra?
   b. Why or why not?

3. What do you think makes someone good at algebra?
   c. Are you good at algebra?
   d. Why or why not?

4. Was the final project in Graded Assignment 6 a fair assessment of what you have learned about linear functions?

5. Which math concepts are you good at understanding?

6. Which math concepts are you not so good at understanding?

7. What are your plans after you graduate high school?

8. Are you planning to take another mathematics class? Which one(s)?

9. Do you use algebra in any other classes? Which ones?

10. Have you used algebra outside of the classroom?

    a. If they respond yes: Please describe what occurred.
    b. If they respond no: Do you feel like you could use algebra outside of the classroom?

11. I will ask the student to find the value that goes in the box and to think aloud to see what they are thinking. $8 + 4 = \square + 5$.

12. Please tell me your definition of a function.

13. Does collecting and graphing real-world data seem useful for determining patterns and making predictions?

14. Why or why not?

15. Is $x - y = z - y$ true always, never, or sometimes? If sometimes, when?

16. Simplify $3a - (b + a)$

17. What number should be in the blank: $2 + 17 = \_\_ + 13$?

18. Find the value of the variable and think out loud, please: $14 - 3z = 8.7 - (-4.3z)$

19. What other comments do you have about algebra and its importance in your life?

Do you have any questions for me?

Appendix F Scope & Sequence of ASK ME-Algebra

Scope and Sequence: Algebra 1, First Semester
Continuing and Extended Education, Distance Education Center
DRAFT

Note: This is a *draft* of the scope and sequence DEC will use to develop the first semester of the Algebra 1 course materials. This Scope and Sequence is based on Wiggins and McTighe's instructional design approach from *Understanding by Design*.

The first stage of design is identifying Desired Results. The Texas Essential Knowledge and Skills (TEKS) are used to identify learning goals and content standards. The national standards published by the National Council of Teachers of Mathematics (NCTM) are also considered, although not explicitly stated. In this document, Desired Results are given in three categories:

- Worth Being Familiar With: "What do we want students to hear, read, view, research, or otherwise encounter?" (page 9)
- Important to Know and Do: What important knowledge and skills must our students have to continue their studies in the field of mathematics?
- "Enduring" Understanding: What do we want our students "to 'get inside of' and retain after they've forgotten many of the details?" (page 10) What do we want them to remember about Algebra five years from now?

During this review process, please provide feedback on the items under "Desired Results". This includes modifying existing items and making recommendations for items that are missing.

The next two phases of design include determining Acceptable Evidence and planning Learning Experiences. Note that the learning experiences will be presented in the context of a complex problem-based theoretical framework.

Lesson 1: Introduction to Functions

Identify Desired Results

Goal/Content Standard: TEKS

b(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
  (A) The student describes independent and dependent quantities in functional relationships.
  (B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.
  (C) The student describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.
  (D) The student represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.
  (E) The student interprets and makes inferences from functional relationships.

b(2) The student uses the properties and attributes of functions.
(B) For a variety of situations the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.
(C) The student interprets situations in terms of given graphs or creates situations that fit given graphs.
b(3) The student understands algebra as the mathematics of generalization and recognizes the power of symbols to represent situations.
(A) The student uses symbols to represent unknowns and variables.

Worth being familiar with

- translate verbal expressions to mathematical expressions
- translate mathematical expressions to verbal expressions
- be familiar with different kinds of data presentation
  be familiar with the idea of recursion and how it is used in mathematics

Important to know and do

- define function, variables (dependent and independent), and dependent relationships among variables in the context of a problem
- describe a reasonable domain and range for a problem situation
- interpret a situation to fit a given graph or create a graph to represent a given situation
- use a graphing calculator to generate a table of data and different kinds of graphs (e.g. scatter plot, linear, etc.) for a function

"Enduring" understanding

- the concept of a function and a solution set
- describing a function using multiple representations, tables, graphs, symbols, and verbal descriptions
- what it means to create and use a mathematical model
- express ideas about functional relationships and concepts
- reflect on the mathematical models created and used

Lesson 2: The Linear Parent Function
Identify Desired Results
Goal/Content Standard: TEKS
b(2) The student uses the properties and attributes of functions. Following are performance descriptions.
(A) The student identifies and sketches the general forms of linear ($y = x$)
(D) In solving problems, the student collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.
b(3) The student understands algebra as the mathematics of generalization and recognizes the power of symbols to represent situations.

(B) Given situations, the student looks for patterns and represents generalizations algebraically.

c(1) The student understands that linear functions can be represented in different ways and translates among their various representations.

(B) The student determines the domain and range values for which linear functions make sense for given situations.

Worth being familiar with

- use appropriate windows and table setup on a graphing calculator to determine graphical and tabular representations of ordered pairs

Important to know and do

- graph ordered pairs on coordinate plane
- Graph the line $y = x$
- use the equation $y = x$ to solve equations graphically
- use the parent function with tables, graphs, algebraic representations to solve problems
- use different strategies to explore data sets and compare these data to the line $y = x$
- use graphs, tables, and algebraic models to identify and interpret the characteristics of a graph (e.g. domain, range, increasing, decreasing, rate of change, y-intercept)

"Enduring" understanding

- Use the line $y = x$ to make predictions and generalizations about a set of data.
- use graphs, tables, and algebraic models to create linear models for a problem situation
- discuss how the characteristics (e.g. domain, range, increasing, decreasing, rate of change, y-intercept) of a linear function change as the algebraic or graphical representation of the parent function changes (specifically identify characteristics of the parent function: constant rate of change of 1, y-intercept of 0, and it's increasing)

Lesson 3: Introduction to Linear Equations

Identify Desired Results

Goal/Content Standard: TEKS

b(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

(B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.

(C) The student describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(D) The student represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(E) The student interprets and makes inferences from functional relationships.

b(2) The student uses the properties and attributes of functions.

(A) The student identifies and sketches the general forms of linear (y = x)

(B) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.
(C) The student interprets situations in terms of given graphs or creates situations that fit given graphs.

b(3) The student understands algebra as the mathematics of generalization and recognizes the power of symbols to represent situations.
(B) Given situations, the student looks for patterns and represents generalizations algebraically.

c(1) The student understands that linear functions can be represented in different ways and translates among their various representations. Following are performance descriptions.
(B) The student determines the domain and range values for which linear functions make sense for given situations.
(C) The student translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

Worth being familiar with

- use the constraints of a problem situation to properly set a graphing calculator window

Important to know and do

- graph ordered pairs on coordinate plane
- solve linear equations graphically
- graph linear functions given a table, symbolic representation or problem situation
- given a set of conditions, use properties of linear functions to determine if a data set represents a function
- use graphs, tables, and algebraic models to identify and interpret the characteristics of a graph (e.g. domain, range, increasing, decreasing, rate of change, y-intercept, minimum, maximum)
- use graphs, tables, and algebraic models (e.g. recursion) to describe linear functions

"Enduring" understanding

- use graphs, tables, and algebraic models to investigate, explore, and generalize patterns in linear functional relationships
- use mathematical symbols, verbal descriptions, tables, and graphs to represent a linear function
- know how different representations can be helpful in order to interpret data

Lesson 4: The Rate of Change and the Y-intercept

Identify Desired Results

Goal/Content Standard: TEKS

b(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. Following are performance descriptions.

(B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.

c(1) The student understands that linear functions can be represented in different ways and translates among their various representations. Following are performance descriptions.

(C) The student translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

c(2) The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

(A) The student develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations.

(B) The student interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

(C) The student investigates, describes, and predicts the effects of changes in m and b on the graph of $y = mx + b$.

(D) The student graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.

(E) The student determines the intercepts of linear functions from graphs, tables, and algebraic representations.

(F) The student interprets and predicts the effects of changing slope and y-intercept in applied situations.

Worth being familiar with

- be familiar with the different kinds of forms used to write linear functions (e.g. point-slope, standard, and slope-intercept)
- justify window choices while using a graphing calculator to represent a problem situation

Important to know and do

- identify the variables represented by an ordered pair and make predictions about the possible values for *x* and *y* based on a set of data
- identify the meaning, if any, of an ordered pair in a problem situation
- solve linear equations graphically
- define *y*-intercept and rate of change or slope
- use algebraic, tabular, and graphical methods and strategies to calculate *y*-intercept and rate of change or slope
- write equations of lines given characteristics such as two points, a point and a slope, or a slope and *y*-intercept
- interpret the meaning of slope and y-intercept in problem situations
- identify the *y*-intercept and rate of change of a linear function given graphs, tables, verbal descriptions and algebraic representations
- use the slope to compare the characteristics of two linear functions (i.e. are they parallel, are they perpendicular)

"Enduring" understanding

- identify the behavior of a function or the data set given the *y*-intercept and the rate of change
- make predictions and generalizations about a problem situation, a set of data, or across sets of data based on the *y*-intercept and/or the rate of change
- understand that if data has a constant rate of change, then it can be modeled with a line and if data does not have a constant rate of change, then you cannot be modeled with a line
- explain how the change in a problem situation or data effects the y-intercept (i.e. explain under what circumstances the *y* value will "shift" up and down the *y*-axis) and the rate of change (i.e. explain under what circumstances the graph becomes more or less steep or increases or decreases)

Lesson 5: Solving Linear Equations

Identify Desired Results

Goal/Content Standard: TEKS

c(3) The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(A) The student analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

(B) The student investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

(C) For given contexts, the student interprets and determines the reasonableness of solutions to linear equations and inequalities.

b(4) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

(B) The student uses the commutative, associative, and distributive properties to simplify algebraic expressions.

Worth being familiar with

- order of operations
- know identities and properties to simplify algebraic expressions
- using concrete models to solve linear equations

Important to know and do

- compare and order rational numbers
- add, subtract, multiply, and divide rational numbers
- apply the following properties to solve linear equations: additive, distributive, multiplicative identity, multiplicative inverse, multiplicative property of zero, reflexive, substitution, symmetric, and transitive
- use algebraic, tabular, and graphing methods and strategies to solve linear equations

"Enduring" understanding

- understand relationships between abstract algebraic models, graphical, tabular, and concrete representations of solving equations
- determine if solution set is reasonable
- make conjectures/hypothesis and provide mathematical support that justifies the strategy used to solve a problem situation

Lesson 6: Inequalities
Identify Desired Results
Goal/Content Standard: TEKS
c(3) The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.
(A) The student analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.
(B) The student investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.
(C) For given contexts, the student interprets and determines the reasonableness of solutions to linear equations and inequalities.

Worth being familiar with

- know properties of inequalities
- using concrete models to solve linear inequalities

Important to know and do

- write the equation for an equality
- graph inequalities on a number line
- apply the following properties to solve inequalities: additive, distributive, multiplicative identity, multiplicative inverse, multiplicative property of zero, reflexive, substitution, symmetric, and transitive
- use algebraic, tabular, and graphing methods and strategies to solve linear equations

"Enduring" understanding

- understand relationships between abstract algebraic models, tables, graphs, and concrete representations of solving inequalities
- determine if a solution set is reasonable
- make conjectures/hypothesis and provide mathematical support that justifies the strategy used to solve a problem situation

Lesson 7: Solving Systems of Equations
Identify Desired Results
Goal/Content Standard: TEKS
c(4) The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(A) The student analyzes situations and formulates systems of linear equations to solve problems.
(B) The student solves systems of linear equations using concrete models, graphs, tables, and algebraic methods.
(C) For given contexts, the student interprets and determines the reasonableness of solutions to systems of linear equations.

Worth being familiar with

none

Important to know and do

- represent a problem situation using a system of linear equations
- use graphing, tables, and multiple algebraic methods and strategies to solve a system of linear equations

"Enduring" understanding

- understand relationships between abstract algebraic models, tabular, graphical and concrete representations of solving systems of equations
- determine if a solution set is reasonable

Lesson 8: Collecting Data and Application Project

Identify Desired Results

Goal/Content Standard: TEKS

b(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
(A) The student describes independent and dependent quantities in functional relationships.
(B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.
(C) The student describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.
(D) The student represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.
(E) The student interprets and makes inferences from functional relationships.

b(2) The student uses the properties and attributes of functions.
(C) The student interprets situations in terms of given graphs or creates situations that fit given graphs.
(D) In solving problems, the student collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

c(1) The student understands that linear functions can be represented in different ways and translates among their various representations.
(A) The student develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations.
(B) The student determines the domain and range values for which linear functions make sense for given situations.

Worth being familiar with

- methods for collecting data
- strategies for choosing a project

Important to know and do

- gather and record data
- identify independent and dependent quantities in the functional relationship.
- write function(s) to describe the data
- use models, tables, graphs, diagrams, verbal descriptions, equations and inequalities to compare and contrast data
- make predictions and generalizations about the data

"Enduring" understanding

- demonstrate the ability to use graphs, tables, and algebraic models to identify and interpret data
- use a modeling process to define a function based on data collected
- use the data to investigate, explore, and generalize patterns in functional relationships

Appendix G Study IRB Approval

Study Number	2004-01-0101	PI Name	Jane C Ries Other Training 07/07/2002
Study Title			
Views about Mathematics			
CO-PI		Other Investigators	
		Shawna K Matteson University of Texas Policies and Procedures Manual 01/27/2005	
Dept Review Contact (DRC)		Faculty Sponsor	
Lawrence D Abraham		Jennifer Smith	

Type of Approval	Approval Start	Approval End
EXPEDITED	02/18/2005	02/18/2006

Vulnerable Populations			
Children			
Risk Level	Minimal risk	Waiver of Consent	No Waiver Specified
Sponsor		OSP Number	

Topics to Address in the Research Proposal

Use this template to provide a description of your research proposal. All applications for review should contain the following information, presented in paragraphs prefaced by the number of the item and the underlined descriptive phrase. When not applicable, DO list the heading and then indicate N/A.

Please note that if this study is part of an NIH funded grant proposal, you will need to attach ONE copy of the complete grant proposal, in addition to the information requested below.

I. Title:

Views about Mathematics

II. Investigators (co-investigators):

Jane Ries, PI
Jenn Smith, faculty advisor

III. Hypothesis, Research Questions, or Goals of the Project:

As it stands now, my researchable question is the following: In what ways could a Problem-Based Instruction (PBI) approach in high school algebra impact students' experiences? Those experiences could include confidence, attitudes, feelings of mastery, enjoyment and engagement.

IV. Background and Significance:

In 1994, the Texas Legislature implemented a policy requiring that "every 9th grade student must be enrolled in algebra I." At a central Texas school in the late 1990's, I noticed that many students would take Algebra I and fail. Most algebra classes had a high failure rate both in the credit for the course and in the End Of Course (EOC) Test. These results were not atypical in Texas. The passing rates given on the Texas Education Agency's web site indicate that the EOC Algebra Test had a high failure rate in all demographic categories across the state.

The main reason could be that the EOC Algebra Test was actually an accurate measure of the students' understanding because of a traditional, direct, procedural method of teaching. With the pressures from administrators for students to pass the EOC Algebra test many teachers felt the need to cover all of the concepts on the EOC Algebra Test and the most efficient way to teach all of the concepts was the lecture method. Research has demonstrated that when teachers are pressured to move through the textbook quickly, understanding of concepts is frequently sacrificed (Battista, 1999; Bransford et al, 2000; Malloy & Malloy,1998). Teaching for understanding requires more time than teachers feel they have.

Therefore, the traditional algebra course the way it is taught now is not effective for a majority of students in the state of Texas. I want to evaluate a method of teaching algebra to high school students that could significantly decrease the failure rate in algebra classes by all students in the state of Texas.

The Online Mathematics Initiative is a research and development effort that investigates a new theoretical model for teaching and learning all mathematics with understanding, which is Problem-Based Instruction (Delisle, 1997; Heid & Zbiek, 1995). The initiative was designed for high school students to work independently. Students who work independently of the school structure should be able to improve their motivation and their time on task since they are not under any time constraints like a class period.

Finding effective ways to use technology and innovative techniques for teaching high school mathematics courses is the emphasis of the Online

Mathematics Initiative. I am participating in the development of a real world problem-based algebra curriculum, which should help improve students' understanding of algebra concepts and attitude towards mathematics. This improvement was noticed in our face-to-face pilot program.

With the use of a survey to measure students' attitude before they start the course, a skills test to determine what they know before taking the course, interviews during the course and another survey and the same skills test at the completion of the course, I should be able to determine that the new method of teaching students algebra concepts is effective for the students enrolled in the course. I would like to see many of the students who start with a negative attitude towards mathematics improve their attitude during the time spent learning the algebra concepts in the new online environment at their own pace.

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V. Research Method, Design, and Proposed Statistical Analysis:

I propose to collect pre- and post-attitude surveys, interviews of a stratified random sample and pre- and post-standardized test scores to investigate the ways in which the curriculum may improve students' attitude towards mathematics and understanding of Algebra concepts.

VI. Human Subject Interactions

A. Identify the **sources of potential participants**, derived materials, or data. Describe the characteristics of the subject population such as their anticipated number, age, sex, ethnic background, and state of health. Identify the criteria for inclusion and/or exclusion.
Explain the rationale for the use of special classes of participants whose ability to give voluntary informed consent may be in question. Such participants include students in one's class, people currently undergoing treatment for an illness or problem that is the topic of the research study, people who are mentally retarded, people with a mental illness, people who are institutionalized, prisoners, etc. When do you expect human subject involvement in this project to begin and when do you expect it to end?

The Distance Education Center (DEC) will recruit the participants since they are researching the same pool of participants. Over the course of the next year DEC is expected to enroll 125 students in the Online Algebra course. I am interested in high school students that enroll in this course. I anticipate high school students' age to be 13 – 19 years old. There are not any filters in place for sex, ethnic background or state of health, so the ratio of participants in these fields should be equal to the ratio of students in the state of Texas.

I will include those participants that are of high school age, 13 – 19, and exclude those that are not in this age range.

Minors might not be able to give voluntary informed consent so their parents will be requested to give informed consent as well.

I expect human subject involvement in this project to begin when the students enroll in the DEC course for the spring of 2004. I expect human subject involvement in this project to end in the spring of 2005.

If the participants are prisoners or residents of correction facilities, the composition of the IRB must be augmented by a prisoner's advocate. Please inform the IRB if this applies to your project**.**

N/A

If some of the potential participants or the parents of child participants are likely to be more fluent in a language other than English, the consent forms should be translated into that language. Both English and the other language versions of the form should be provided, with one language on one side of a page and the other on the other side of the page. This translation may be completed after IRB approval of the study and consent forms. Specify here your intentions with respect to the languages of the consent forms. (If you plan to conduct your study with students from the Austin Independent School District, you will be required to provide a Spanish language version of your parental consent form.)

I plan to offer consent form with both English and Spanish with one language on one side of a page and the other language on the other side of the page.

B. Describe the **procedures for the recruitment of the participants**. Append copies of fliers and the content of newspaper or radio advertisements. If potential participants will be screened by an interview (either telephone or face-to-face) provide a script of the screening interview.

The Distance Education Center (DEC) will recruit students for their course and research and I will use the data they collect from those students.

If the potential participants are members of a group that may be construed as stigmatized (e.g., spousal abusers, members of support groups, people with AIDS, etc.) your initial contact with the potential participants should be through advertisements or fliers or through people who interact with the potential participants because of their job duties. These people may describe your study to the potential participants and ask them to contact you if they are interested in talking to you about the study.

N/A

C. Describe the **procedure for obtaining informed consent**.
If you do not plan to obtain active written consent, specifically point this out and explain why not. Include the consent form(s) for review. Minors (people under 18) need parental consent to participate in studies. Participants between 7 and 17 should be given an opportunity to assent to their participation. (See Sample Assent Forms for Minors).

Consent forms will be obtained and stored by DEC. The forms (with a copy for the participants to keep) will be mailed to the participants and then mailed back to DEC.

D. <u>Research Protocol</u>. What will you ask your participants to do? When and where will they do it? How long will it take them to do it? Describe the type of research information that you will be gathering from your subjects, i.e., the data that you will collect. *Append copies of all surveys, testing materials, questionnaires, and assessment devices. Append copies of topics and sample questions for non-structured interviews and focus group discussions.*

Students will take the survey as part of the course. Which means they will fill it out online – at their own computer - and submit their responses to the server at DEC. The interviews will be conducted over the telephone and recorded then transcribed. This is the survey that will be given to the students before they start the course and after they complete the course: See Appendix.

The interview will be questions pertaining to how the course's structure is helping/hindering their understanding of the algebra concepts (e.g. Does the Mathpad applet help or hinder your understanding of graphing linear equations?). There will also be questions about how their attitude may or may not be changing (e.g. Has the recycle project helped you to see that mathematics can be used outside of the classroom setting?). The last type of question would be for the students to explain how they are currently feeling about mathematics.

VII. Describe any **<u>potential risks</u>** (physical, psychological, social, legal, or other) and assess their likelihood and seriousness. Describe alternative and potentially less risky methods, if any, that were considered as possible methods and why they were not used. If the research methods impose risks on the subjects, include evidence that may justify their use (such as previous experience with the procedures). Most studies pose some degree of risk, even though the risk may be minimal. For example, one common risk is the loss of the confidentiality of the participants' responses.

One potential risk is the loss of the confidentiality of the participants' responses.

One risk that may arise in studies with children or interviews with parents about their children, is the risk that you may acquire information about familial child abuse. If you acquire this information, you are required to report it to Child and Family Protective Services, 1-800-252-5400. If your study is likely to result in responses that may suggest child abuse and you do not provide anonymity to the respondents, you must inform the parents in the consent form (and the child in the assent form) that you are legally required to report this information. Research data can be subpoenaed by a court of law, so questions about illegal activities such as drug use place respondents at risk unless the participants' responses are anonymous. A Certificate of Confidentiality can eliminate the risk of having one's data subpoenaed.

N/A

Describe the procedures for protecting against (or minimizing) any potential risks and include an assessment of their effectiveness. Discuss the procedures that will be used to maintain the confidentiality of the research data. If the subject's responses are taped and the tape can be linked to a participant because his or her name is on an audiotape or because the tape is a videotape, precautions must be taken. These safeguards include storing the tape in a secure place (file cabinet in a locked office), limiting access to the tape to the researcher and his or her associates, and destroying the tape, if it is reasonable to do so, after it has been transcribed or the information on it has been coded. Describe the disposition of the tapes in the consent form. If the tapes are to be retained after the study is completed and they have been analyzed, explain the rationale for doing so in the proposal and state that they will be retained in the consent form.

The survey results will be stored on the Distance Education Center's server where only the researchers will have access to the data. The interviews will be taped and then transcribed. The tapes will be stored at DEC where only the researchers will have access to them. The tapes will be destroyed when it is reasonable to do so.

If the study involves a procedure that introduces a physical risk, specify arrangements for providing medical treatment if it should be needed. If the study involves a procedure that introduces a psychological risk, such as the recall of a traumatic event, specify arrangements for providing psychological treatment if it should be needed. Please state whether or not you will provide payment for physical or psychological harm if it is incurred.

N/A

If your study involves deception, describe the procedures for debriefing the participants.

N/A

VII. Describe and assess the **potential benefits** to be gained by participants (if any) and the benefits that may accrue to society in general as a result of the planned work. Discuss the risks in relation to the anticipated benefits to the participants and to society.

The students will be given the opportunity to complete Texas' Algebra I course and potentially receive credit towards graduation from their high school. One risk is that the students don't complete the course.

IX Indicate the specific **sites or agencies involved in the research project** besides The University of Texas at Austin. These agencies may include school districts, day care centers, nursing homes, etc. Include, as an attachment, approval letters from these institutions or agencies on their letterhead. The letter should grant you permission to use the agency's facilities or resources; it should indicate knowledge of the study that will be conducted at the site. If these letters are not available at the time of IRB review, approval will be contingent upon their receipt.

N/A

X. If the project has had or will receive **review by another IRB**, indicate this. Attach a copy of this approval to this application or submit it to the IRB secretary of the IRB when you receive it. The UT IRB will usually accept the versions of consent forms that have been approved by IRBs affiliated with hospitals or medical schools or by the site where the research will be conducted.

N/A

Appendix H "Learnability" of Mathematics

5. Learning mathematics requires (a) a serious effort (b) a special talent.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 19: "Learnability" Question 5

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	3	3	1	4	4	5	1
Post	4	4	5	1	1	4	1
Quest. 5 1, 2, 3: Expert 4, 5: Mixed	From Expert to Mixed	From Expert to Mixed	From Expert to Mixed	From Mixed to Expert	From Mixed to Expert	Stayed Mixed	Stayed Expert

9. For me, doing well in mathematics classes depends on (a) how much effort I put into studying (b) how well the teacher explains things in class.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 20: "Learnability" Question 9

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	2	5	5	2	4	4	4
Post	4	4	7	4	4	4	4
Quest. 9 1, 2, 3: Expert 4: Mixed	From Expert to Mixed	From Naïve to Mixed	Stayed Naïve	From Expert to Mixed	Stayed Mixed	Stayed Mixed	Stayed Mixed

11. When studying mathematics in a textbook or in class materials (a) I memorize it the way it is presented (b) I make sense of the material so that I can understand it.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 21: “Learnability” Question 11

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	2	3	3	5	4	5	4
Post	2	2	6	4	7	4	4
Quest. 11 5, 6, 7: Expert 4: Mixed	Stayed Naïve	Stayed Naïve	From Naïve to Expert	From Expert to Mixed	From Mixed to Expert	From Expert to Mixed	Stayed Mixed

13. In mathematics, it is important for me to (a) memorize technical terms and mathematical formulas (b) understand the ideas and when and how to use them.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 22: “Learnability” Question 13

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	6	2	3	4	3	4	2
Post	2	3	6	4	3	4	3
Quest. 13 4, 5, 6, 7: Expert 3: Mixed	From Expert to Naïve	From Naïve to Mixed	From Mixed to Expert	Stayed Expert	Stayed Mixed	Stayed Expert	From Naïve to Mixed

19. How well I do on mathematics exams depends on how well I can (a) recall material in the way it was presented in class (b) do tasks that are somewhat different from ones I have seen before.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 23: “Learnability” Question 19

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	2	2	3	1	1	7	4
Post	2	6	Blank	4	1	4	5
Quest. 19 6, 7: Expert 3, 4, 5: Mixed	Stayed Naïve	From Naïve to Expert	Stayed Mixed	From Naïve to Mixed	Stayed Naïve	From Expert to Mixed	Stayed Mixed

Appendix I Personal Relevance

6. If I had a choice: (a) I would never take another mathematics class (b) I would still take mathematics for my own benefit.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 24: Personal Relevance Question 6

Name	Ana	Bob	Bubba	Barbara	Jean	Nan	Vince
Pre	1	4	2	4	1	5	4
Post	4	1	1	1	5	4	4
Quest. 6 6, 7: Expert 4, 5: Mixed	From Naïve to Mixed	From Mixed to Naïve	Stayed Naïve	From Mixed to Naïve	From Naïve to Mixed	Stayed Mixed	Stayed Mixed

27. For me, the relationship of mathematics classes to everyday life is usually (a)easy to recognize (b) hard to recognize.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 25: Personal Relevance Question 27

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan M.	Vince
Pre	4	7	3	3	5	1	5
Post	3	7	6	4	3	6	4
Quest. 27 1, 2, 3, 4: Expert 5: Mixed	Stayed Expert	Stayed Naïve	From Expert to Naïve	Stayed Expert	From Mixed to Expert	From Expert to Naïve	From Mixed to Expert

36. I study mathematics: (a) to satisfy graduation requirements (b) to learn useful knowledge.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 26: Personal Relevance Question 36

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan M.	Vince
Pre	2	2	5	5	2	7	4
Post	5	3	1	4	6	4	5
Quest. 36 6, 7: Expert 4, 5: Mixed	From Naïve to Mixed	Stayed Naïve	From Mixed to Naïve	Stayed Mixed	From Naïve to Expert	From Expert to Mixed	Stayed Mixed

Appendix J Role of Reflective Thinking

10. When I experience a difficulty while doing mathematics (a) I quickly seek help, or give up trying (b) I try hard to figure it out on my own.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 27: Role of Reflective Thinking Question 10

Name	Ana	Bob	Bubba	Barbara	Jean	Nan	Vince
Pre	2	7	3	1	4	4	3
Post	4	1	4	4	3	4	7
Quest.10 6, 7: Expert 5: Mixed	Stayed Naïve	From Expert to Naïve	Stayed Naïve	Stayed Naïve	Stayed Naïve	Stayed Naïve	From Naïve to Expert

17. In order to solve a mathematics problem (a) I need to have seen the solution to a similar problem before (b) I apply general problem solving techniques.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 28: Role of Reflective Thinking Question 17

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan M.	Vince
Pre	2	1	3	3	6	4	2
Post	6	5	4	4	5	4	4
Quest. 17 4, 5, 6, 7: Expert 3: Mixed	From Naïve to Expert	From Naïve to Expert	From Mixed to Expert	From Mixed to Expert	Stayed Expert	Stayed Expert	From Naïve to Expert

18. Seeing alternate solutions to a mathematics problem is (a) a waste of my time (b) helpful for improving my problem solving abilities.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 29: Role of Reflective Thinking Question 18

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan M.	Vince
Pre	6	6	3	3	3	6	5
Post	5	7	1	6	3	5	4
Quest. 18 6, 7: Expert 5: Mixed	From Expert to Mixed	Stayed Expert	Stayed Naïve	From Naïve to Expert	Stayed Naïve	From Expert to Mixed	From Mixed to Naïve

24. For me, making unsuccessful attempts when solving a mathematics problem is (a) a natural part of my pursuit of a solution to the problem (b) an indication of my weaknesses in mathematics.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 30: Role of Reflective Thinking Question 24

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	6	2	5	5	3	6	5
Post	6	4	6	7	4	5	5
Quest. 24 1, 2: Expert 3: Mixed	Stayed Naïve	From Expert to Naïve	Stayed Naïve	Stayed Naïve	From Mixed to Naïve	Stayed Naïve	Stayed Naïve

Appendix K Structure and Knowledge of Mathematics

14. The purpose of mathematical formulas is to (a) express meaningful relationships among variables (b) provide ways to get numerical answers to problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 31: Structure and Knowledge of Mathematics Question 14

Name	Ana	Bob	Bubba	Barbara	Jean	Nan	Vince
Pre	4	4	3	4	5	4	5
Post	4	3	4	4	4	4	4
Quest.14 1, 2, 3: Expert 4: Mixed	Stayed Mixed	From Mixed to Expert	From Expert to Mixed	Stayed Mixed	From Naïve to Mixed	Stayed Mixed	From Naïve to Mixed

16. The first thing I do when solving a word problem that involves mathematics is (a) represent the situation with sketches and drawings (b) search for formulas that relate givens to unknowns.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 32: Structure and Knowledge of Mathematics Question 16

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	6	5	3	4	2	6	4
Post	7	4	4	6	5	5	4
Quest. 16 1, 2, 3: Expert 4: Mixed	Stayed Naïve	From Naïve to Naïve	From Expert to Mixed	From Mixed to Naïve	From Expert to Naïve	Stayed Naïve	Stayed Mixed

23. Collecting and graphing real world data is useful for (a) determining patterns and making general predictions (b) obtaining numerical answers to specific problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 33: Structure and Knowledge of Mathematics Question 23

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan	Vince
Pre	4	7	3	7	3	4	6
Post	1	3	4	4	4	6	3
Quest. 23 1, 2, 3: Expert 4, 5: Mixed	Mixed to Expert	From Naïve to Expert	From Expert to Mixed	From Naïve to Mixed	From Expert to Mixed	From Mixed to Naïve	From Naïve to Expert

25. When solving a challenging mathematics problem, a mathematician (a) makes many incorrect attempts (b) moves directly to a correct solution.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 34: Structure and Knowledge of Mathematics Question 25

Name	Ana M.	Bob B.	Bubba	Barbara	Jean	Nan M.	Vince
Pre	6	6	5	5	2	6	3
Post	4	4	6	4	4	5	4
Quest.25 1, 2: Expert 3: Mixed	Stayed Naïve	Stayed Naïve	Stayed Naïve	Stayed Naïve	From Expert to Naïve	Stayed Naïve	From Mixed to Naïve

37. After I have answered all questions in a homework mathematics problem: (a) I stop working on the problem (b) I check my answers and the way I obtained them.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 35: Structure and Knowledge of Mathematics Question 37

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	2	5	3	6	1	5	5
Post	4	4	1	4	4	4	6
Quest. 37 6, 7: Expert 5: Mixed	Stayed Naïve	From Mixed to Naïve	Stayed Naïve	From Expert to Naïve	Stayed Naïve	From Mixed to Naïve	From Mixed to Expert

38. After the teacher solves a mathematics problem for which I got a wrong solution: (a) I discard my solution and learn the one presented by the teacher (b) I try to figure out how the teacher's solution differs from mine.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 36: Structure and Knowledge of Mathematics Question 38

Name	Ana M.	Bob	Bubba	Barbara	Jean	Nan M.	Vince
Pre	4	3	5	4	4	4	5
Post	4	4	2	3	1	4	3
Quest. 38 6, 7: Expert 5: Mixed	Stayed Naïve	Stayed Naïve	From Mixed to Naïve	Stayed Naïve	Stayed Naïve	Stayed Naïve	From Mixed to Naïve

41. Scientists use mathematics as: (a) a tool for analyzing and communicating their ideas (b) a source of factual knowledge about the natural world.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 37: Structure and Knowledge of Mathematics Question 41

Name	Ana M.	Bob B.	Bubba	Barbara	Jean	Nan M.	Vince
Pre	4	4	3	1	6	5	4
Post	1	4	4	7	4	7	6
Quest.38 1, 2, 3: Expert 4: Mixed	Mixed to Expert	Stayed Mixed	Expert to Mixed	Expert to Naïve	From Naïve to Mixed	Stayed Mixed	From Mixed to Naïve

Appendix L Validity of Mathematical Knowledge

8. My score on mathematics exams is a measure of how well (a) I understand the covered material (b) I can do things the way they are done by the teacher or in some class materials.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 38: Validity of Mathematical Knowledge Question 8

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan M.	Vince
Pre	5	3	2	4	5	6	3
Post	5	4	1	1	blank	4	3
Quest. 8 1, 2, 3, 4: Expert 5: Mixed	Stayed Mixed	Stayed Expert	Stayed Expert	Stayed Expert	Stayed Mixed	From Naïve to Expert	Stayed Expert

15. After I go through a mathematics textbook or class materials and feel that I understand them (a) I can solve related problems on my own (b) I have difficulty solving related problems.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 39: Validity of Mathematical Knowledge Question 15

Name	Ana M.	Bob B.	Bubba	Barbara	Jean C.	Nan M.	Vince
Pre	2	7	3	2	6	6	6
Post	3	5	6	4	4	4	5
Quest. 15 1, 2: Expert 3: Mixed	From Expert to Mixed	Stayed Naïve	From Mixed to Naïve	From Expert to Naïve	Stayed Naïve	Stayed Naïve	Stayed Naïve

26. The process of attempting to solve a problem that involves mathematical reasoning is (a) a satisfying experience (b) not a satisfying experience.

Only (a)	Mostly (a)		equally (a) & (b)		Mostly (b)	Only (b)
1	2	3	4	5	6	7

Table 40: Validity of Mathematical Knowledge Question 26

Name	Ana M.	Bob B.	Bubba	Barbara	Jean	Nan M.	Vince
Pre	6	2	5	4	4	5	4
Post	3	7	7	7	5	6	3
Quest.26 1, 2: Expert 3, 4: Mixed	From Naïve to Mixed	From Expert to Naïve	Stayed Naïve	From Mixed to Naïve	From Mixed to Naïve	Stayed Naïve	Stayed Mixed

Appendix M TEKS Objectives

TAKS 9th grade Objective 1: Functional Relationships

TEKS Algebra 1

(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations.

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

TAKS 9th grade Objective 2: Properties and Attributes of Functions

TEKS Algebra 1

The student uses the properties and attributes of functions. The student is expected to:

(B) **identify the mathematical domains and ranges and determine reasonable domain and range values for a variety of situations.**

(C) **interpret situations in terms of given graphs or create situations that fit given graphs.**

TAKS 9th grade Objective 4: Linear Equations and Inequalities

TEKS Algebra 1

(3) The student formulates equations and inequalities (based on linear functions), uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems.

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solve the equations and inequalities.

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities for given contexts.

(4) The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods.

TEKS 8th Grade

(3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

(A) compare and contrast proportional and non-proportional relationships; and

(B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.

(4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, **equation,** or verbal description.

(5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

(A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and **algebraic equations**; and

(B) use an algebraic expression to find any term in a sequence.

TAKS 7th grade Objective 2: Patterns, Relationships and Algebraic Reasoning

TEKS 7th grade:

(3) Patterns, relationships, and algebraic thinking. The student solves problems involving proportional relationships. The student is expected to:

(B) estimate and find solutions to application problems involving **proportional relationships such as similarity, scaling, unit costs**, and related measurement units.

(4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:

(A) **generate formulas** involving conversions, perimeter, area, circumference, volume, and scaling;

(5) Patterns, relationships, and algebraic thinking. The student uses equations to solve problems. The student is expected to:

(A) **use concrete models to solve equations and use symbols to record the actions; and**

TAKS 8th grade Objective 1: Numbers, Operations, and Quantitative Reasoning

TEKS 8th Grade

(1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

(B) select and use appropriate forms of rational numbers to solve real-life problems **including those involving proportional relationships**;

(2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

(D) **use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, a = 1.4h.**

TAKS 9th grade Objective 3: Linear Functions

TAKS Algebra 1
(2) The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. The student is expected to:
(D) The student graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.

TAKS 9th grade Objective 10: Mathematical Processes and Tools

TEKS 8th Grade
(15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, **or algebraic mathematical models**; and

(B) evaluate the effectiveness of different representations to communicate ideas.

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From: "Jarrell, Dick" <Dick.Jarrell@tea.state.tx.us>
Date: March 27, 2006 1:04:58 PM CST
To: "Jane Ries Cushman" <jane.ries@alumni.utexas.net>
Subject: Permission to use TAKS Materials-Jane Ries Cushman/Doctoral Candidate

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To: Jane Ries Cushman
Doctoral Candidate
The University of Texas at Austin
College of Education
Mathematics and Science Education
(512)232-2279 wk
(512)282-5842 hm

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Vita

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